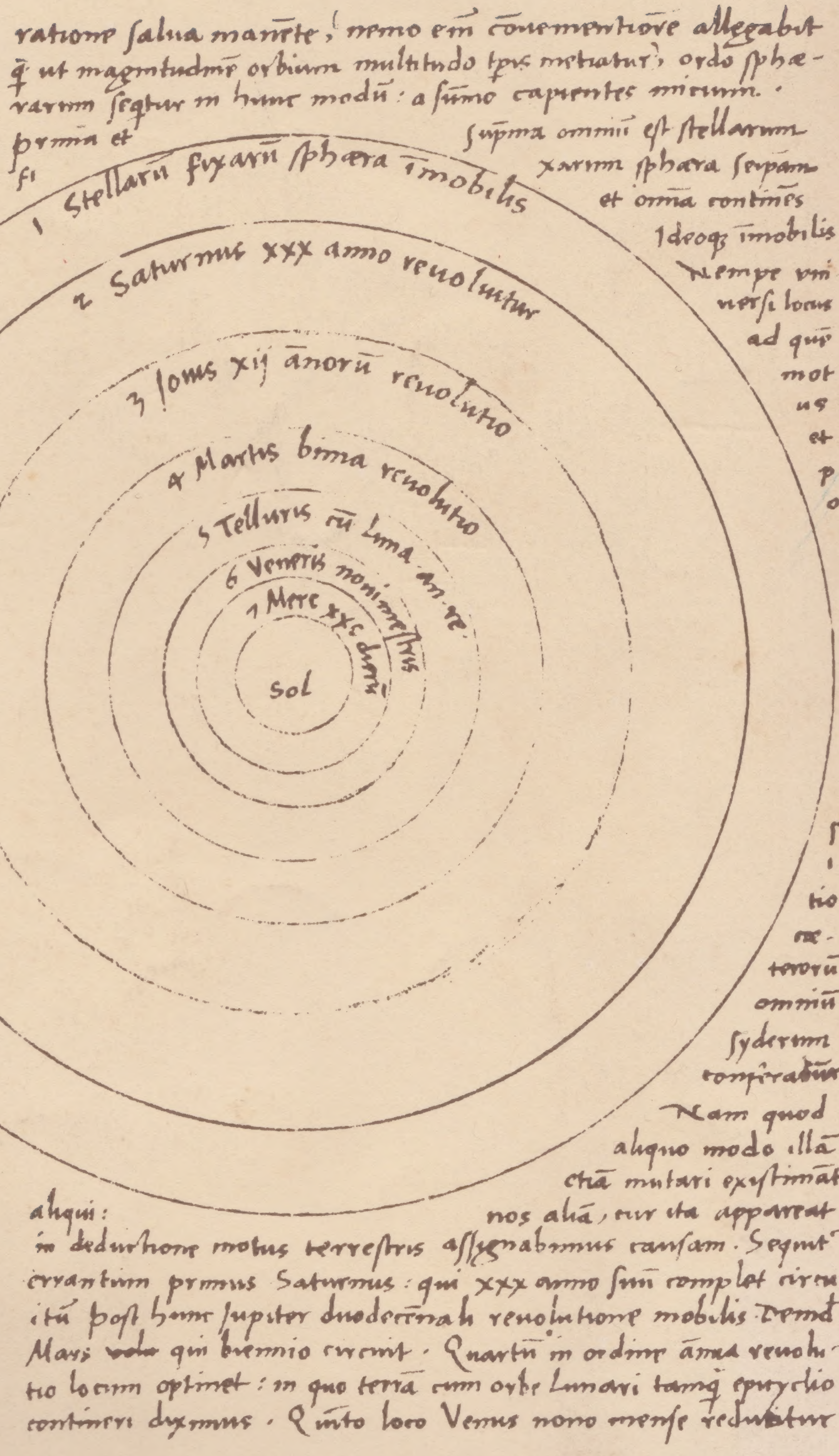
S101 Units 2 and 3

THE OPEN UNIVERSITY

Science : A Foundation Course

2 Measuring the Solar System



3 Motion Under Gravity

A Scientific Theory

The Open University
Science: A Foundation Course

Unit 3
Motion under gravity: a scientific theory

Prepared by the Science Foundation Course Team

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SCIENCE

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Study guide

This Unit has two main components, the Main Text and the television programme, TV 03. Here are some points to bear in mind in studying the Unit.

The Main Text

The Home Experiments are woven into the text. Don't be tempted to skip them and use the results in the Appendices; one of the main aims of this Unit is to introduce experimental and mathematical skills you will need later in your studies and the experiments are designed to give you practice in acquiring these skills.

Part of the text is concerned with *arguing the case* for Newton's laws, but if you look at the Objectives listed at the end of the Unit you will find that an ability to reproduce these arguments is given little emphasis. So don't grapple endlessly with the justification for Newton's laws. If you find the ideas too difficult to grasp fully in a reasonable time, don't worry. The arguments are difficult and it is better that you should concentrate on the conclusions, Newton's laws, than waste too much time on the supporting details.

In reading the text you will find frequent marginal references to the supporting *MAFS* blocks.* In writing this Unit we have allowed time for you to follow up these references, and so, if you have any doubts about your mathematical skills, nail them *now*. Leave them and you are sure to have trouble later.

The TV programme

You will find that you will gain most benefit from the programme if you have already studied up to the end of Section 4. So, at the least, try to find time to read up to this point before the programme is broadcast.

Part of the programme is devoted to momentum, a subject only touched on in the Main Text. Momentum is an important quantity that you will need later in the Course—don't treat this part of the programme as optional or superfluous.

* The Open University (1978) S101 *Mathematics for the Foundation Course in Science* (*MAFS*), The Open University Press.

'Nature and Nature's laws lay hid in night.
God said "let Newton be" and all was light'.
(Alexander Pope, *Epitaph*)

1 Introduction

This Unit begins to ask the question *why*. Here are some *why*-questions.

Why do apples fall to the ground?

Why does the Moon go round the Earth rather than the Sun?

Why are some things heavier than others?

Why are some things harder to push than others?

Why do planets move in ellipses?

Why are Kepler's laws obeyed?

All these questions are about motion and it is to the general question, '*Why* do things move?', that this Unit is addressed.

Many of the basic ideas and terms we now use to describe motion were first formulated in the fifteenth, sixteenth and seventeenth centuries. Questions of the type we have asked above were of great philosophical, religious or practical importance. For example, in the Roman Catholic Church, which was undergoing a crisis of authority, religious and scientific dogma had become interwoven and descriptions of planetary motion were held to have implications for man's view of God. The military establishment needed to understand the principles governing the motion of 'projectiles'. The importance of the subject of motion was reflected in the talents of the people drawn into working towards an understanding of its nature. Among those who made significant contributions were Leonardo da Vinci, Copernicus, Tycho Brahe, Kepler, Galileo, Descartes and many others. Their efforts finally bore fruit in 1687 when Isaac Newton published the book widely known as his *Principia*, 'The Mathematical Principles of Natural Philosophy'. In this book he set out three universal laws of motion and a theory of universal gravitation*. The framework of ideas established by Newton allows us to answer all the questions listed above.

This Unit will attempt to justify Newton's ideas. The development is not chronological—rather we shall try to show the plausibility of his ideas and demonstrate that their success comes from Newton's careful synthesis of mathematically exact definitions and physical intuition. The precise use of language is of particular importance in this area of study where so many of the words (force, mass, weight, inertia, gravity, density, etc.) are used in common speech. It is one of the hallmarks of a scientific theory that words are used in specialized and well-defined ways. Try to be as careful in your use of these terms as Newton was.

In Sections 2 and 3 we shall be concerned with laying the foundations for the Unit; the terms velocity, acceleration, mass and force will be discussed, and, through these, Newton's first two laws will be explained. Section 4 uses these basic definitions to throw light on the phenomenon of gravity, and shows that mass is a gravitational parameter**.

After this fairly lengthy exercise in logic and deduction, Section 5 uses Home Experiments to investigate the gravitational force. Some of the experiments are

* You might wonder why we refer to the three *laws* of motion and the *theory* of gravitation. Don't worry about this distinction; the choice of the word *laws* follows that conventionally adopted. Nevertheless after reading this Unit you may feel that, in as much as these 'laws' introduce new scientific concepts, they would together be more correctly described by the more dignified term, 'theory of motion'.

** A parameter is a number that can be attached to something to identify how it will respond in a certain situation. So, mass is a property of an object that determines how it responds to the phenomenon of gravity.

entirely home-based; in others, there are practical difficulties in such an approach and instead we have included the data from 'laboratory' experiments in the text. The analysis of these 'raw results' is left for you to do. If you have any difficulty with the experiments or analysis, Appendices 1–7 provide the information demanded from you in the text. Don't be tempted to use the Appendices and skip the experiments: working the results out for yourself is an excellent way of clarifying the difficult ideas presented in this Unit. Look upon each Appendix as an SAQ answer; look at each one *after* you have attempted to work out your own results. Section 5 ends with the form of Newton's universal law of gravitation. In the final part of the Unit, Section 6, Newton's laws are used to explore the internal composition of our planet, the Earth, and its Moon.

2 Motion

To follow Newton's explanation of planetary motion, you will find it necessary to be quite clear in your understanding of how motion is described. The scientific vocabulary is important in this respect, and so in this Section the meaning of the relevant terms will be developed and defined.

2.1 Velocity as a measure of instantaneous motion

We will start with the word 'motion' itself. Quite simply, motion is the process by which the position of an object changes with time. But how is this motion to be described?

> Does the announcement 'the train standing at platform three will be going to Glasgow', completely specify the future motion of the train?
>
> Clearly it doesn't. The announcer has omitted to mention when, by what route and how quickly the train is to go to Glasgow.

Here is another, less trivial, example of an object moving. A ship sails from its home port and radios back its position at noon each day. Its progress is shown in Figure 1. The home port is represented as being at the intersection of the graph's axes, the origin 0. The axes define directions, east and north. At noon on the first day after leaving port the ship has reached a position (A) which is 50 miles east and 40 miles north of its home port. A day later it has reached position B which is 170 miles east and 200 miles north of its home port.

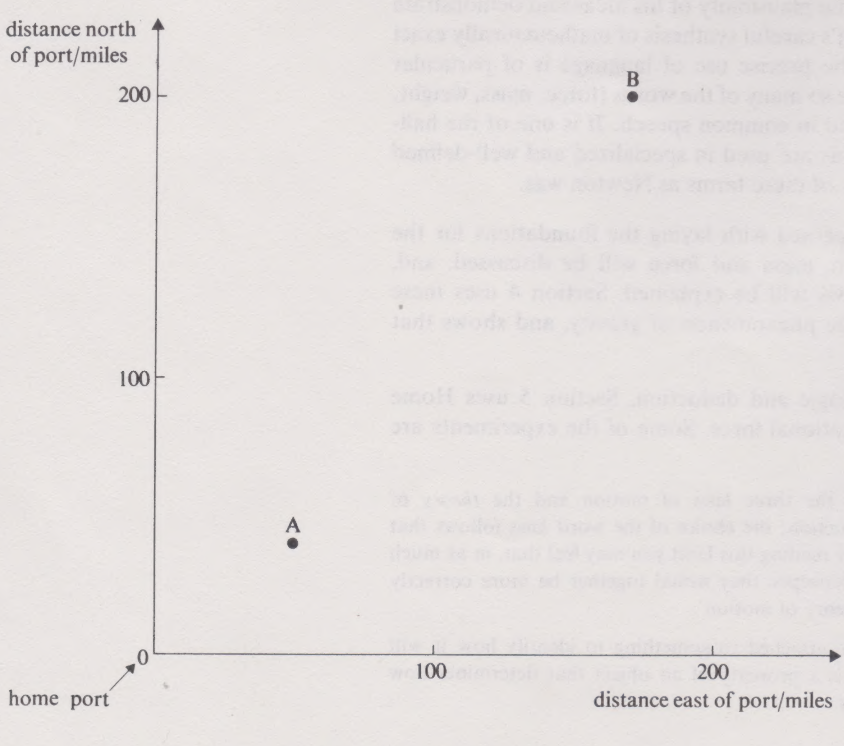


FIGURE 1 A and B are the positions of a ship at noon on the first two days after leaving port.

The two positions provide some clues to the motion of the ship. For instance it is possible to work out the distance between points A and B.

One method of doing this is to use a relationship that the Greek philosopher Pythagoras proved to exist between the lengths of the sides of any triangle with one of its angles a right angle (that is, 90° or $\pi/2$ radians). Such a triangle is shown in Figure 2. The relationship is:

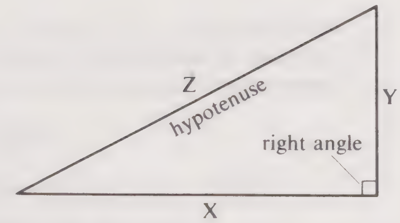


FIGURE 2 Pythagoras's theorem: $Z^2 = X^2 + Y^2$

> The sum of the squares of the lengths of the sides enclosing the right angle is equal to the square of the length of the side opposite the right angle (the hypotenuse), that is:
>
> $$Z^2 = X^2 + Y^2 \quad \text{(in Figure 2)} \tag{1}$$

Pythagoras's theorem

If you are unsure about the details of this theorem, you will find it helpful to take a look at Block 4 of MAFS where it is proved and examples of its use are given.

MAFS 4

From this geometrical relationship it is straightforward to work out the length of the line AB. Draw a vertical line through B and a horizontal line through A to form a right-angled triangle.

SAQ 1 What is the distance between the positions A and B?

SAQ answers begin on page 46.

So, the distance 'as the crow flies' between points A and B can be found quite easily. Of course the ship and the crow might have followed completely different routes, and, as the crow's route is the shortest, the distance actually travelled by the ship might be much more than 200 miles.

> What do we know about the speed of the ship during the 24 hour period?

Speed is the rate at which an object traverses a distance, that is:

speed

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}} \tag{2}$$

Unfortunately we can't directly apply this formula. All we know is that the ship has travelled at least 200 miles. We don't know exactly how far it has gone, or whether it changed speed during the day. However, all is not lost; we can say for certain that, on average, the speed of the ship was sufficient for it to travel at least 200 miles in 24 hours. Its average speed was at least $200/24$ ($= 8\frac{1}{3}$) miles h^{-1}*.

MAFS 1

This doesn't really help us much in our quest to find out about the motion of a ship during the day. For a better idea of how it was moving at some *instant* we need to know its *instantaneous speed*, the rate at which it was traversing a distance at that *instant*. This might sound a difficult idea but in fact it is instantaneous speed we mean when we use the word speed in common speech. A 30 m.p.h (miles h^{-1}) speed limit forbids instantaneous speeds greater than 30 m.p.h; it is no defence to claim that your average speed was less than the limit.

> What other information is needed to describe the instantaneous motion of the ship?

With a full description of the motion, it should be possible to work out where the ship was just after the moment at which its motion was known. But knowing the speed alone is not sufficient to allow us to do this: we also need to know in which direction it was moving at that moment. *The speed in a known direction is called the velocity*. The distinction between the two quantities, speed and velocity, may sound unimportant to you, and the tendency is to use the words

velocity

* The units we are using here are miles per hour, that is, the number of miles travelled divided by the number of hours taken. This division is recognized by the use of notation involving a negative exponent in miles h^{-1}. Check Block 1, Section 1.4.6, of *MAFS* if you're unsure about this.

interchangeably. Try to avoid doing this. It is often important to take the direction of motion into account, as you will see later in this Unit, and the correct use of the terms then helps the clarity of the scientific argument.

$$\text{velocity} = \text{speed} + \text{direction} \tag{3}$$

SAQ 2 Which of the following statements is true?

(a) If you know the velocity of an object you know its speed.

(b) If you know the speed of an object you know its velocity.

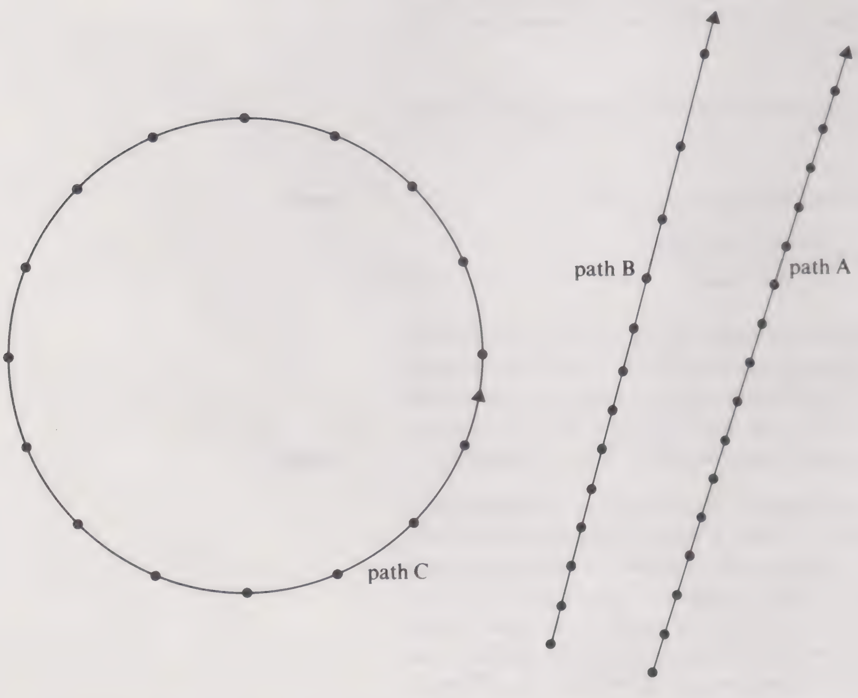
SAQ 3 A yacht sails at constant speed directly towards a buoy 500 metres to the north of its starting position. If it arrives there in 100 seconds, what was its velocity?

2.2 Acceleration as rate of change of velocity

Look at Figure 3. You will see three curves consisting of dots joined by lines. The lines are the paths of three objects. The dots are the positions of each object at closely and equally separated instants of time. They are meant to convey to you the velocity of the object: the greater the velocity, the longer the line between adjacent dots. In this way the curves each represent the motion of an object.

In which of the three paths is the velocity constant?

FIGURE 3



You probably had no difficulty deciding that the object following path A has constant velocity. The dots are evenly spaced on a straight line and therefore the magnitude (the numerical value) and the direction of the velocity are both constant.

Again, path B is fairly simple to understand. The dots are initially evenly spaced, but about halfway along the path they start to spread out, signifying that the velocity is increasing. The velocity is therefore changing with time, a process we call *acceleration*. In just the same way that velocity describes the rate of change of the position of an object with time, *acceleration describes the rate of change of velocity with time*, that is:

acceleration

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken for velocity to change}} \tag{4}$$

SAQ 4 What are the dimensions of acceleration? What are its units in the SI system?

SAQ 5 An oil-tanker slips anchor and accelerates in a straight line. After 10 minutes it has reached a speed of $5\,\mathrm{m\,s^{-1}}$. What was its average acceleration over this period?

SAQ 6 While travelling with a velocity of $5\,\mathrm{m\,s^{-1}}$ in a narrow shipping lane, an emergency occurs and, throwing the engines into reverse, the captain stops the tanker without changing course. If it takes 5 minutes for the ship to come to rest, what is its average acceleration?

In path C (Figure 3) the *velocity is not constant*, even though the dots are evenly spaced, implying that *the speed is constant*. The velocity is changing because the direction of motion is changing.

Is the object following path C accelerating?

It may seem odd to you, but the answer is yes, even though the speed is constant. The conclusion follows from the definition of acceleration as rate of change of velocity. The object following path C is changing its velocity: it is therefore accelerating.

The difficulty can be overcome quite simply, by realizing that acceleration has a direction attached to it, just as velocity does. In the oil-tanker examples the acceleration acted either in the same direction as the velocity or in the opposite direction, and there were no complications. In path C, however, the speed is constant and therefore the acceleration at any instant cannot be acting along the same line as the velocity. If it were the speed would change. In fact it must be acting in a direction *perpendicular* to the instantaneous direction of motion.

You might like to draw on Figure 3 lines that are perpendicular to the path of the object at a few points round the circle. The lines you have drawn should lie along the radii of the circle: the object's path is consistent with an acceleration which, at any instant, acts towards the centre of the circle. But the bending produced by the acceleration is insufficient for the object ever to reach the centre. Instead, it follows a circular path.

Don't worry too much at this stage if you find these ideas difficult. We shall be returning to them later.

SAQ 7 Why must we say that all athletes accelerate as they go round the last lap of a 10 000 metre race?

2.3 What needs to be explained about motion?

Our reasons for wanting a theory of motion are quite clear: we want to be able to answer the questions posed at the beginning of the Unit. But, where do we start? What is so obvious as not to require explanation? Is there a fundamental observation about 'the nature of motion' that everybody agrees on? The assumptions shape the theory and it is worth spending some time examining the background to Newton's choice of assumptions.

First, we will look back at the way the philosophers of the Aristotle school analysed the problem of motion. They framed their theory in terms of 'natural motion'. They believed that any body spontaneously moved back towards its natural resting place. Heavy bodies, which were made of the 'elements', earth and water, fell in a straight line to the Earth, while light bodies (made of the other two elements, fire and air) rose to the sky, their natural resting place. These were 'natural motions'. Celestial bodies were considered to be entirely different from terrestrial bodies: their 'natural motion' was along a circular path and they therefore moved in orbits. Aristotle argued that any deviation from natural motion was *forced*, requiring some external agency.

It is tempting to pour scorn on these apparently primitive ideas, but that would be most unfair. The views held by the Aristotelian school contain the kernels of many of the ideas we now accept: elements, natural motion, external agencies. Naturally, though, their scientific thinking did not exist in isolation from their society as a whole and, as happens today, philosophical and social attitudes influenced the science.

When Newton was seeking to construct a theory of motion he was able to adopt a different standpoint. That he was able to do so was largely due to Galileo, who died in the year that Newton was born. In a careful series of experiments, Galileo had investigated the motion of balls rolling down inclined planes. One experiment in particular is of interest. He allowed a ball to roll down one inclined plane and then back up another (Figure 4). By taking great care to reduce friction in his experiments he was able to convince himself that the ball would, in the ideal experiment, roll up to the same level from which it started. The amount of guesswork involved in this conclusion was in fact quite small; Galileo was an ingenious experimentalist and his experiments approximated quite closely to the frictionless ideal. Furthermore, this conclusion was not altered by changing the slope of the second plane. The ball always rolled up to a height equal to the starting height. Galileo then argued to the extreme case when the second plane had zero slope, that is, when it was horizontal.

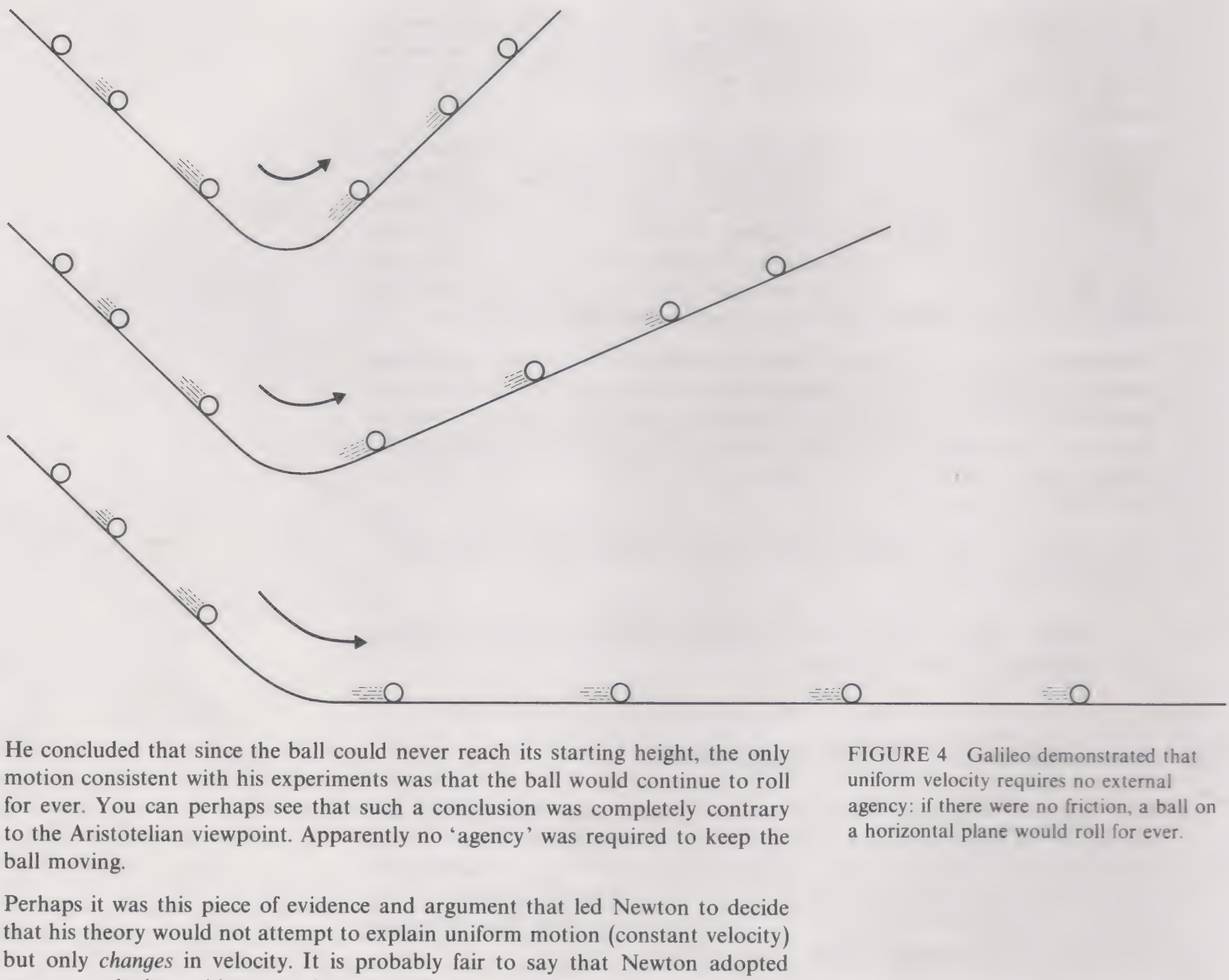


FIGURE 4 Galileo demonstrated that uniform velocity requires no external agency: if there were no friction, a ball on a horizontal plane would roll for ever.

He concluded that since the ball could never reach its starting height, the only motion consistent with his experiments was that the ball would continue to roll for ever. You can perhaps see that such a conclusion was completely contrary to the Aristotelian viewpoint. Apparently no 'agency' was required to keep the ball moving.

Perhaps it was this piece of evidence and argument that led Newton to decide that his theory would not attempt to explain uniform motion (constant velocity) but only *changes* in velocity. It is probably fair to say that Newton adopted constant velocity as his 'natural motion'. To make this approach quite explicit he advanced a general principle—that every object will remain at rest (that is, with constant velocity = 0) or continue with constant velocity unless something causes it to accelerate.

2.4 Force as a cause of acceleration

You are quite probably unimpressed by this statement. After all there is no way in which it can be shown to be false. If we see an object at rest, we say there is nothing causing it to accelerate. If we see it moving with constant velocity, likewise. If, on the other hand, it is accelerating, we merely assert that there must be a cause. Why is the statement so important?

Examine it carefully and you will see that in formulating the statement (which is essentially his first law of motion) Newton made one vital contribution: he

asserted that *all accelerations are caused by something.* For historical reasons the something is given the name *force*. That is all Newton has done so far. He has not tried to explain uniform motion. Why matter should behave in this way is just as obscure as it ever was. Soon you will see that this modest first step leads to greater things.

force

The concept of force is defined scientifically by Newton's first law as that which causes acceleration and the definition does correspond to some extent with common usage. For instance, you might say 'I had to force my son to go to bed last night'. If you physically pushed him up the stairs Newton would not argue with your choice of words. If however you said 'I am having to force myself to keep reading', it would possibly be true but it would certainly be an unscientific use of the word 'force'.

SAQ 8 Are there any forces acting on the Moon?

In real life there is often a complication which obscures the connection between acceleration and force. For instance try to answer the following question:

Do you have to apply a force to push a car at constant velocity along a flat road?

FIGURE 5 The car moves with constant velocity if the force provided by the pusher is exactly balanced by frictional forces.

Your experience probably tells you that you do, but Newton would seem to be suggesting that, because the car is not accelerating, no force is needed. What we have neglected so far is that more than one force can act on the same object, as is shown in Figure 5 for the case of the car. When the car is moving at constant velocity, frictional forces are exactly balancing the muscular force of the pusher. There is no *net force* and therefore no acceleration. On the other hand, when you are getting it going, the 'push' is bigger than the resistance and there is a net force. The net force produces an acceleration and the car starts moving.

net force

SAQ 9 Why doesn't a chair accelerate downwards when you sit on it?

From this argument, you might think that balanced forces have no effect on an object. Not so! There is another process which an object subject to balanced forces can undergo without changing its velocity, namely deformation. A rubber ball can be squeezed by the action of two equal and opposite forces without it accelerating. In fact this property in which balanced forces deform objects can be used as the basis of a scale of forces.

2.5 Newton's first law of motion

Every body continues in a state of motion with constant velocity unless acted on by some unbalanced force.

Newton's first law

2.6 Objectives of Section 2

After reading this Section you should be able to:

(a) Differentiate between speed and velocity (SAQ 3).

(b) Calculate the velocity of an object, given the time taken for it to cover a known distance with constant velocity (SAQ 2).

(c) Calculate the acceleration of an object, given the time taken for it to change its velocity by a known amount while travelling in a straight line (SAQs 5 and 6).

(d) Explain why an object following a curved path is of necessity accelerating (SAQ 7).

(e) State Newton's first law of motion and use it to explain motion qualitatively in simple situations (SAQs 8 and 9).

You should also be able to:

(f) Use the negative exponent notation in writing the units of velocity and acceleration (SAQ 4).

(g) Use Pythagoras's theorem to calculate the length of one side of a right-angled triangle, given the lengths of the other two (SAQ 1).

3 Motion, mass and force

In the previous Section the rather loose ideas we all have about the words used to describe motion began to be brought into a scientific framework—the terms were defined. At the end of the Section we put a name to whatever it is that brings about acceleration. We called it a force. But a very basic problem remains: we can describe the motion which results from the application of an unbalanced force but we have no idea how to relate the acceleration of the object to the force it is experiencing. Is there some property of the object that determines this relationship? This question is the subject matter of Section 3.

3.1 Why is it harder to push a lorry than a car?

Ask somebody why it is harder to push a lorry than a car and the most common replies you are likely to receive are 'it's bigger' and 'it's heavier'. But are these properties of 'bigness' and 'heaviness' definable and measurable in the same way that for example length can be measured? In other words, can they be used in a scientific theory of motion as properties that influence that motion in a consistent way?

Tentatively, the reply 'it's bigger' might mean one of two things: the lorry has a larger volume than the car, or the lorry has more matter in it than the car.

There are two black-painted blocks on a table. They are outwardly identical (as in Figure 6). Is it possible to say without touching them which is easier to push?

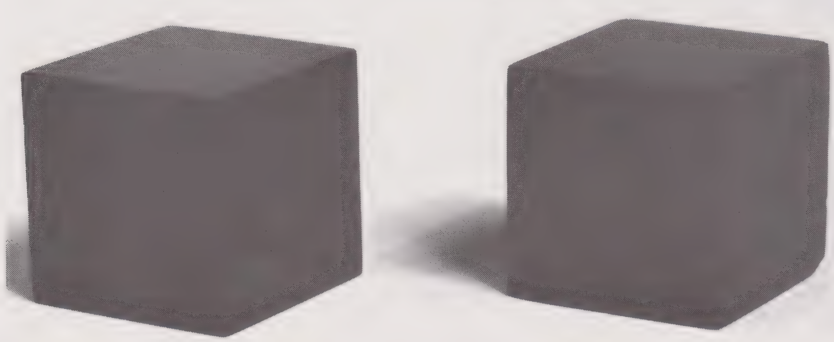


FIGURE 6 Which block is made of iron? You can only tell by moving them.

Obviously the answer is no. In fact the one on the left is made of plastic and is much easier to push than the one on the right which is made of iron. It seems that the volume of an object, which is measurable, does not relate in a simple way to motion. It needs to be combined with a knowledge of what the object is made from.

Is there 'more matter' in a block of plastic or a block of iron of identical size?

At first sight it is tempting to say that the iron has more matter in it, but before doing so we ought to be clear what we mean by 'more matter'. Is there a 'matter scale' which is as obvious and intuitively acceptable as say a 'length scale'? Nobody has thought of one so far even though many distinguished minds in the Middle Ages tried. In fact the 'explanation' that the lorry has 'more matter' than the car is just a different way of saying that the lorry is harder to push than the car. The 'explanation' is, at best, a restatement of the observation!

Is an object that is heavier always harder to push?

'Heaviness' does seem to be potentially a more useful property in that how hard an object is pulled towards the Earth seems to be related to how hard it is to push. What is more, it is possible to construct a scale of heaviness using a spring in that the heavier the object the more a spring supporting it stretches. The length of the spring defines the 'heaviness' of the object. This property of springs is exploited in laboratory spring balances.

Unfortunately there is a major insurmountable snag in such a scale. We are looking for a property that we can attach to an object (for example, 'Object A has 3 units of heaviness') but if you take an object to the North Pole you will find that it is pulled downwards a little harder than it was at the Equator. The effect, the spring stretches a bit more, is small but detectable. If you could take an object to the Moon you would find its 'heaviness' was six times smaller! 'Heaviness' is *not* an internal property of an object: it depends on where it is in relation to other objects (for example, the Earth). Our attempts to explain the relative ease of pushing a car compared to a lorry lie in ruins. It seems that there is a confusion of properties which relate *in some way* to an object's motion but are stubbornly difficult either to quantify or define.

Newton cut through this confusion. He realized that it was not possible to find a motion-influencing property inherent to an object that could be measured separately from its motion. Instead he *defined* a property which was measurable *in terms of the motion.*

3.2 Mass

The property Newton defined was *mass*. He asserted that mass is a measure of how difficult it is to accelerate an object, and that it could be found by detecting its effects on motion. A simple, idealized experiment will show you how Newton managed to perform this apparent 'sleight of hand'. **mass**

Imagine two trolleys, A and B, connected by a string. We will assume that the trolleys are frictionless and are so light that they have no effect on the outcome of the experiment. Obviously these assumptions can never be completely correct but you would probably be surprised how closely they can be fulfilled. A compressed spring is placed between the trolleys and two brass cubes of identical size and shape are placed on the trolleys (Figure 7a, p. 16). One cube is placed on A and one on B. The string is then cut, the compressed spring extends, and the trolleys accelerate in opposite directions. At some instant while the trolleys are accelerating (i.e. before the spring is completely extended), the accelerations of A and B are measured.

How do you think the accelerations of A and B will be related?

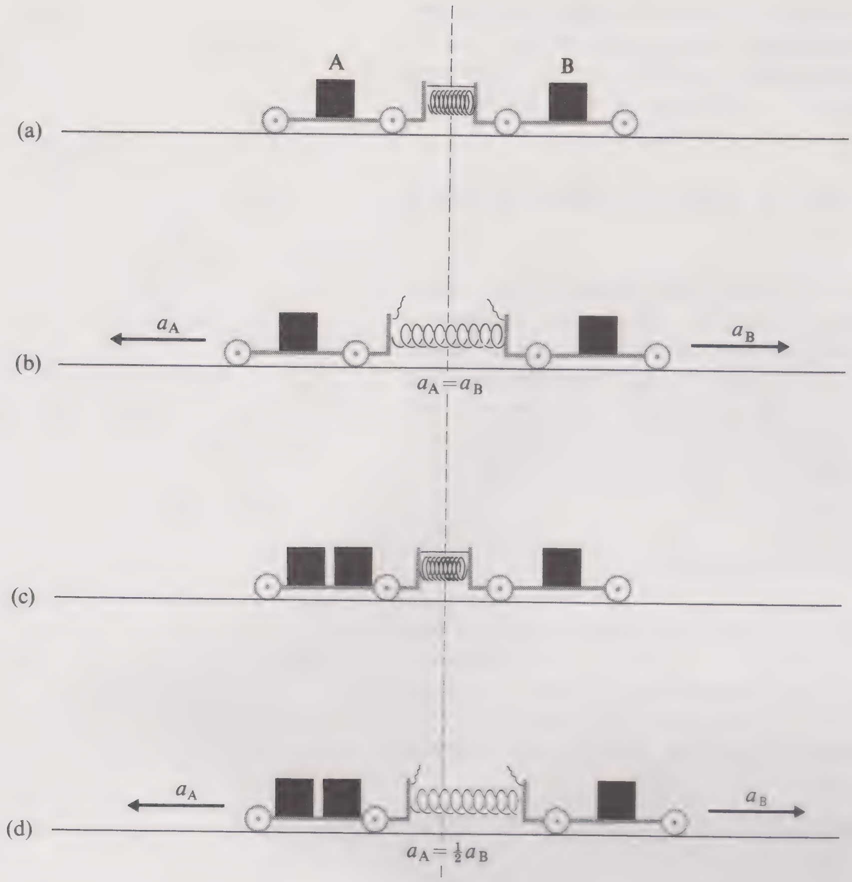


FIGURE 7 By comparing the accelerations of different objects a scale of mass can be defined. Full discussion in text.

By the symmetry of the experiment your answer should be that the accelerations will be equal but in opposite directions (see Figure 7b). So far the experiment may seem trivial but it can be made more interesting by repeating it with two brass cubes on A and one on B (as in Figure 7c).

How do you think the instantaneous accelerations will now be related?

The answer to this question isn't at all obvious. In fact the acceleration of A is found to be half that of B (Figure 7d). Notice that doubling the number of cubes halves the acceleration. At this stage it would be worth checking that the choice of spring or cube is not affecting the outcome of the experiment. We could try it again with the original spring turned round, with *any* other spring or with two of the cubes swapped round, but the result would *always* be the same. The trolley with two cubes would have half the acceleration of the trolley with one cube.

Can you predict what will happen if the experiment is again repeated with three cubes on A and one on B?

Correct, the acceleration of A will be one-third that of B. There is a definite mathematical relationship between the acceleration and the number of blocks.

By this experiment it is possible to define the property we call mass. It is the quantity that governs the relative acceleration of the two trolleys (and their contents). If the mass of A is m_A and the mass of B is m_B, *we can define mass by the equation*

$$\text{mass of A} \times \text{acceleration of A} = \text{mass of B} \times \text{acceleration of B} \qquad (5)$$

In symbolic form,

$$m_A a_A = m_B a_B \qquad (6)$$

where a_A and a_B are the instantaneous accelerations of A and B. There is nothing mysterious about this definition. After all, if mass is some internal property of an object we would expect three cubes of brass to have three times the mass of one cube of brass. The experiment has demonstrated that the acceleration of the three cubes is one-third that of the single cube. So, the equation defining mass fits in with our common sense.

SAQ 10 What will be the relative accelerations in a trolley and spring experiment if the mass of one trolley and its contents is very much larger than that of the other?

A *scale of mass* is now relatively easy to establish. It is only necessary to *choose* an object and *define* it to have one unit of mass. As you have already read in Unit 2 (Section 1.4), in the International Bureau of Weights and Measures there is a platinum–iridium cylinder which is *defined* to have one unit of mass, the kilogram (abbreviated kg). The mass of any other object can be measured (in principle if not in practice) by placing the standard kilogram on say trolley A and the object of unknown mass on trolley B and repeating the experiment. The accelerations of the two trolleys are measured and, from equation 5 or 6:

$$\text{unknown mass} \times \text{its acceleration} = \text{standard kg} \times \text{its acceleration}$$

that is,

$$\text{unknown mass} = \left(\frac{\text{acceleration of standard kg}}{\text{acceleration of unknown mass}}\right)\text{kg} \quad (7)$$

3.3 A scale of force

Let us review what has been achieved. In Section 3 we are aiming to relate the acceleration of an object to the force acting on it via some property of the object itself. So far, we have managed to establish a property, mass, that certainly influences acceleration but, as yet, the connection between force and mass and acceleration remains obscure.

The qualitative role of force (as something that causes accelerations) was explained in Newton's first law, but now it must be defined quantitatively in such a way that it can be measured.

The mass determination experiment, in which two objects were propelled by the same spring (Figure 7), hinted at how this can be done. The experiment showed that the product of mass and acceleration for one object was the same as for the other object. Perhaps Newton took this as his cue in proposing a definition of force—*the product of mass and acceleration.*

The proposal is quite reasonable. Look at Figure 8, which shows the implications of this proposal. Twice as much force will accelerate twice as much mass by the same amount, or alternatively, the same mass by twice as much. It is even more to the point that, by defining force in this superficially acceptable way, a verifiable and consistent theory of motion can be established. Objects have consistent masses in different experiments; springs compressed by the same amount produce the same force on different masses. Experiment is the ultimate test of theory and therefore, as the experiments confirm the theory, Newton's definition of force is acceptable.

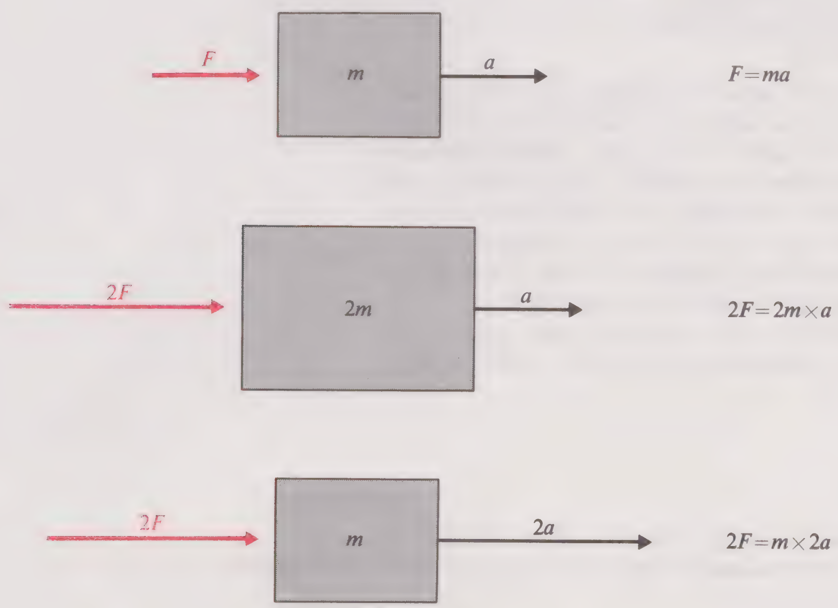


FIGURE 8 Newton defined force as the product of mass and acceleration: (a) $F = ma$. (b) Twice as much force will accelerate twice as much mass by the same amount, or (c) the same mass by twice as much.

3.4 Newton's second law of motion

Newton's ideas on the relationship between the concepts of force, mass and motion are made explicit in his second law:

Newton's second law

A force of magnitude F causes a body of mass m to accelerate in the direction of the force with an acceleration of magnitude a according to the equation:

$$F = ma \tag{8}$$

newton (N)

Appropriately, the unit of force in the SI system is the *newton* (abbreviated to the single letter N). One newton is that force which accelerates a mass of 1 kg by $1\,m\,s^{-2}$, that is:

$$1\,N = 1\text{ newton} = 1\,kg\,m\,s^{-2} \tag{9}$$

What are the dimensions of force?

Remember from Unit 2 that equations can only equate like with like. The dimensions of force are therefore those of mass times acceleration, namely $(\text{mass}) \times (\text{length}) \times (\text{time})^{-2}$, that is

$$\begin{aligned}[\text{force}] &= [M] \times [L] \times [T^{-2}] \\ &= [MLT^{-2}]\end{aligned}$$

SAQ 11 In SAQ 5 you found that an oil-tanker accelerated at $(1/120)\,m\,s^{-2}$. If the mass of the tanker is 1.2×10^8 kg, how large is the accelerating force?

SAQ 12 How quickly would a force of the magnitude that accelerated the tanker be able to stop a 1 000 kg car moving with a speed of $20\,m\,s^{-1}$ (approximately 40 m.p.h.)? *Hint* First work out the acceleration and then, knowing the initial speed, the time taken to stop.

You are probably wondering why so much fuss is made about Newton's first two laws. Their power lies in the way they scientifically define the hitherto vague concepts of force and mass. In Newton's framework, scales of force and mass can be established through unambiguous relationships with the intuitively acceptable concepts of length and time. Motion in the Newtonian universe arises from forces acting on masses and, once these quantities are known, all future motion can, in principle, be calculated. Shortly you will be exploring how these ideas are applied to gravitational motion, but first there are two points that need to be remarked on.

The first is quite simple. In the statement of Newton's second law the phrase 'accelerates in the direction of the force' was used. This is commonsense: it recognizes that force, like acceleration and velocity, has a directional quality. If you push an object forwards it doesn't go sideways. So when you specify a force, make sure you specify its direction.

On a more esoteric note, there is an assumption in Newton's second law which is not immediately apparent. Mass has been defined in terms of acceleration and no consideration has been given to the speed of the object. Newton assumed that the result of any mass determination experiment is not affected by the speed of the object. Quite a reasonable assumption, you might think. It is in fact wrong, but only in certain very special circumstances. In the early 1900s experiments proved that mass increased with speed in the way predicted by Einstein's theory of relativity. Fortunately the effect of increasing mass is only detectable at extremely high speeds—even Concorde, at full speed, increases its mass by less than one part in 10^{11}. Newton's picture of the world is quite adequate for most purposes.

3.5 A short aside on momentum

momentum

Newton's second law can be expressed in a slightly different way by introducing a quantity called *momentum*. For everyday material objects the momentum p is the product of the object's mass m and velocity v.

$$p = mv \qquad (10)$$

Remember that Newton's second law is $F = ma$ and that acceleration is defined as change in velocity $(v_2 - v_1)$ divided by time, that is:

$$a = (v_2 - v_1)/t \qquad (11)$$

We can replace the symbol a in $F = ma$ by the expression in equation 11:

$$F = \frac{m(v_2 - v_1)}{t} = \frac{mv_2 - mv_1}{t} \qquad (12)$$

Of course $(mv_2 - mv_1)$ is just the change in momentum $(p_2 - p_1)$. So, Newton's second law can be expressed as:

force = change in momentum per unit time

$$F = \frac{p_2 - p_1}{t} \qquad (13)$$

Although we shall not need the concept of momentum in this Unit (it is particularly useful in analysing collisions between objects), you will meet it again later in the Course and in the TV programme associated with this Unit (TV 03).

3.6 Objectives of Section 3

After reading Section 3 you should be able to:

(a) State a definition of mass in terms of the equation $m_A a_A = m_B a_B$ and the experiment to which it refers (SAQ 10).

(b) State Newton's second law of motion.

(c) Calculate the net force on an object, the acceleration of the object, or its mass, given two of the three quantities (that is, apply $F = ma$) (SAQs 11 and 12).

4 Gravity and mass

In the previous Section you learnt how the property mass could be defined in a way that allowed the ideas of force and acceleration to be brought together into a mathematical equation: Newton's second law. This is often referred to as mass acting as *inertia*—the bigger the mass, the more difficult it is to move. But mass has a dual role. As well as being the property describing the inertia of an object, it is also the property that determines how the object interacts with other objects via the phenomenon of *gravity*.

inertia

gravity

4.1 The force of gravity

Two systems that betray the infiuence of gravity should be familiar to you: the apple falling to the Earth and the Moon orbiting the Earth (Unit 1). How do Newton's ideas on motion help to explain the behaviour of these systems?

When an apple falls from a tree, it falls downwards! It *accelerates* downwards and according to Newton there must therefore be a downward force acting on it. Wherever the orchard is (Figure 9, p. 20), the apple falls towards the centre of the Earth. Does this suggest to you that there is an attractive force between the apple and the Earth itself?

gravitational force

The Moon is in orbit round the Earth. Although it doesn't appear to be falling in the same way as the apple, it is following a circular path and therefore, by the argument of Section 2.2, it is accelerating in a radially inward direction. The acceleration immediately implies that a force is acting in the direction of this acceleration. Does the fact that the Moon orbits the Earth once again suggest to you that the origin of the force is the Earth itself?

Newton had the insight to realize that the forces acting on the Moon and the apple are of the same type: both are manifestations of the force of gravity by which *all masses attract all other masses.* Don't be tempted to think that Newton explained the cause of the force of gravity. He didn't*. But he showed that it operates in the same way for the Sun, Earth, Moon, planets, apple and indeed everything else that can be measured.

FIGURE 9 Falling apples accelerate along the radii of the Earth; they are attracted towards the centre of the Earth.

4.3 Free-fall—a home observation

An intriguing feature of the gravitational force can be revealed by a very simple Home Experiment.

Choose two objects—one heavy and one light. Historical precedent would suggest that you choose either cannonballs (like Galileo) or apples (like Newton) but these are respectively in short supply and easily bruised. A 10p and $\frac{1}{2}$p piece will do the job just as well. Hold them at the same height above the ground and release them at the same time. Which hits the ground first?

We hope you haven't skipped this experiment. The result isn't obvious. They hit the ground at the same time. Even if you repeat the experiment several times, you won't find the 10p piece falling more quickly. Now this 'falling together' is surprising, so you should check for yourself whether or not a wider range of objects also fall at the same rate.

Balance two or three objects (a pen, ruler, etc.) on this book and then pull away the book sharply downwards. Did all the objects hit the ground at the same time? You may uncover one of the difficulties in this type of experiment, depending on your choice of object. For instance a feather or a sheet of paper will fall more slowly than say a rubber. The complication is that an object like a feather has a small mass but a large surface area. So it is difficult for the feather to move through the air because there is air resistance to take into account.

If you were able to repeat this experiment in a vacuum, where there is no air, you would find that *all* objects, including feathers, would fall at the same rate

* To be more precise, Newton did not *at first* attempt to explain the cause of gravity. In later years, he abandoned his principle of 'feigning no hypotheses' by putting forward a very speculative theory for the origin of the gravitational force.

(Figure 10). They would undergo *free-fall*. In TV 03 a spectacular version of this experiment is performed by the first American astronauts to land on the Moon. There is no atmosphere on the Moon and therefore no air resistance.

free-fall

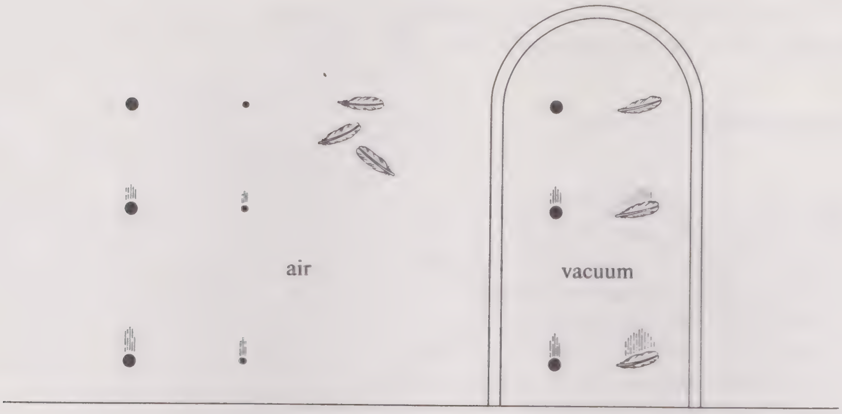


FIGURE 10 In a vacuum *all* objects have the same acceleration due to gravity.

Ignoring these slight complications, the main message of your experiment should be clear. All objects, irrespective of their masses and compositions, take the same time to fall the same distance. *They accelerate under the force of gravity at the same rate.* The importance of this conclusion is difficult to overstate: it is the key to understanding the second half of this Unit.

4.3 Mass as gravitational response—weight

The conclusion of the previous Section can be restated as:

> The acceleration due to gravity is independent of the mass of the falling object.

That leads on to a question of fundamental importance.

> Is the accelerating force due to gravity also independent of the mass?

Newton's second law says that it is not. The force is found by multiplying the mass of an object by its acceleration. But the acceleration due to gravity is the same for each object even though different objects have different masses.

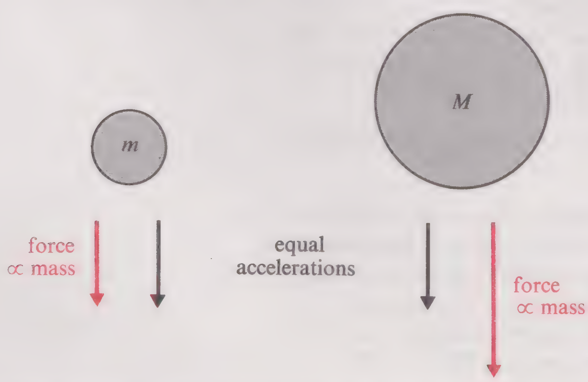


FIGURE 11 For the small object the accelerating force is equal to its small mass times its acceleration. For the larger mass the accelerating force is equal to its larger mass times the *same* acceleration. The conclusion is that the accelerating force is proportional to the mass of the object.

The only way for both of these statements to be true is for the force of attraction of an object to the Earth to be proportional to its mass, with the constant of proportionality* equal to the acceleration due to gravity (Figure 11). In symbol form,

MAFS 1

$$F = ma \quad (14)$$

$$= m \times (\text{constant})$$

Therefore $$F \propto m \quad (15)$$

* There is a useful discussion of proportionality in Block 1 of *MAFS* if you are unsure about the language used in this statement.

We have deduced something quite remarkable; mass is not only the property that determines the inertia of an object; it is also the property that determines the *gravitational force* the object feels.

Now, the gravitational force between the Earth and, for instance, a ball is just the *weight* of the ball. But equation 15 tells us that this force is proportional to the mass of the ball. Therefore the weight of an object is proportional to its mass, that is:

weight

$$\text{weight} = \text{mass} \times \text{acceleration due to gravity} \tag{16}$$

Therefore
$$\text{weight} \propto \text{mass} \tag{17}$$

You may find this proportionality confusing. The vital thing to remember is that the mass of an object is a property of the object itself and is therefore constant (except at very high speeds) but the weight depends on where it is. The weight is a *force* of attraction between one object and another object (usually the Earth) and that force is different in different places. The confusion between mass and weight comes through in common speech in a comment such as 'I weigh 86 kilograms'. The kilogram is the unit of mass; weight is a force and should strictly speaking be expressed in newtons. You might try filling in your next medical form in newtons and see what the reaction is.

> **SAQ 13** We commented earlier that the weight of an object depends on where it is. In fact, the weight of an object at the North Pole is 0.5 per cent more than its weight at the Equator. Will the rate at which it accelerates, when falling freely, be the same at the two places?

4.4 Mass as gravitational cause

Already you have seen that the force of gravity links two objects; the *apple* is attracted to the *Earth*, but, as yet, we have not investigated what property of the Earth influences this attraction. To find a possible answer to this question you must look to another system, the solar system, in which the components are linked by the force of gravity. Be warned though that the reasoning leading to the answer is going to be much more subtle than the observation that starts the argument.

A paradox: The planets including the Earth go round the Sun in fairly smooth elliptic orbits. Therefore the Sun is the major influence on the Earth's motion. The Moon goes round the Earth in a smooth orbit. It appears to be tied to the Earth and, by the same argument, the Earth is the major influence on the Moon's motion. Now, think about this question. If the Earth is the major influence on the Moon's motion, why isn't the converse true: why isn't the Moon the major influence on the Earth's motion? Rephrasing the question, why doesn't the Moon affect the Earth as much as the Earth affects the Moon?

A possible answer: Perhaps the inequality in their behaviour is the result of the Earth being bigger than the Moon.

Now that answer seems reasonable, but it is only a guess, and, what is more, an ill-defined guess. Scientific reasoning must be based on precise statements. So we shall translate this guess into a precise statement using the terms we have so far developed, and see if it throws any light on the problem. The assumption is that the Earth accelerates the Moon more than the Moon accelerates the Earth in proportion to the ratio of the masses of the Earth and the Moon, that is:

$$\frac{\text{acceleration of Moon}}{\text{acceleration of Earth}} = \frac{\text{mass of Earth}}{\text{mass of Moon}}$$

In symbolic form,
$$\frac{a_M}{a_E} = \frac{M_E}{M_M} \tag{18}$$

The implications of the equation can be made much clearer by rearranging its mathematical form. Multiplying both sides by the mass of the Moon and the acceleration of the Earth gives

$$a_M M_M = a_E M_E$$

$$\begin{matrix}\text{acceleration} \\ \text{of Moon}\end{matrix} \times \begin{matrix}\text{mass of} \\ \text{Moon}\end{matrix} = \begin{matrix}\text{acceleration} \\ \text{of Earth}\end{matrix} \times \begin{matrix}\text{mass of} \\ \text{Earth}\end{matrix}$$

Each side of the equation has a familiar ring to it—mass times acceleration is, of course, force. Therefore,

$$\text{force on Moon due to Earth} = \text{force on Earth due to Moon} \tag{19}$$

The inequality we started with has disappeared. If our assumption is true, the accelerations of the Moon and Earth are different, but the forces on them are not (Figure 12). *The Moon exerts a force on the Earth that is exactly equal (but opposite) to the force that the Earth exerts on the Moon.* That seems fair, but what implications does it have for the mathematical form of the gravitational force, in particular its dependence on mass, which is after all what we are trying to find?

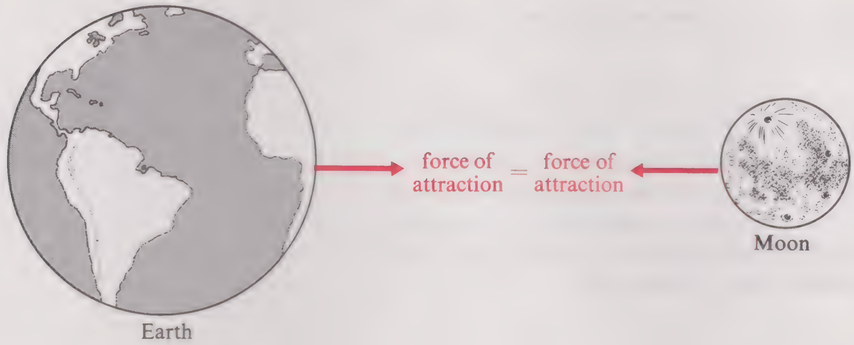


FIGURE 12 The force attracting the Earth to the Moon is equal (but opposite) to the force attracting the Moon to the Earth.

Well, in Section 4.3 we showed that the gravitational force acting on an object was proportional to its mass. So, for the Earth and the Moon,

$$\begin{aligned} &\text{gravitational force on Moon} \propto \text{mass of Moon} \\ \text{and} \quad &\text{gravitational force on Earth} \propto \text{mass of Earth} \end{aligned} \tag{20}$$

Now we believe that the forces on the Earth and Moon, the left-hand sides of these two expressions, are the same force (equation 19). If that is so, then this force, the mutual attraction of the Earth and Moon, must be proportional to *both* the mass of the Earth *and* the mass of the Moon, that is:

$$\text{force of attraction } \textit{between} \text{ Earth and Moon} \propto \begin{matrix}\text{mass of}\\ \text{Earth}\end{matrix} \times \begin{matrix}\text{mass of}\\ \text{Moon}\end{matrix} \tag{21}$$

To reach this conclusion, you will remember, we made an assumption that explained qualitatively the orbital behaviour of the Earth and Moon. How can we be sure that the assumption, and therefore conclusion, are both valid? Quite simply, the conclusion fits experiments. As we have already remarked, the ultimate test of a theory lies in experiment and we now know that the general result—that the gravitational force of attraction between any two objects is proportional to the product of their masses—is consistent with both the motion of our solar system and terrestrial measurement.

The mass of an object, it seems, is not only a measure of its own inertia and of its own gravitational response to the presence of another material object: it is also the cause of the gravitational force that the object exerts on other objects. In symbolic form, for the two masses, m_1 and m_2 in Figure 13,

$$F_{\text{attraction}} \propto m_1 m_2 \tag{22}$$

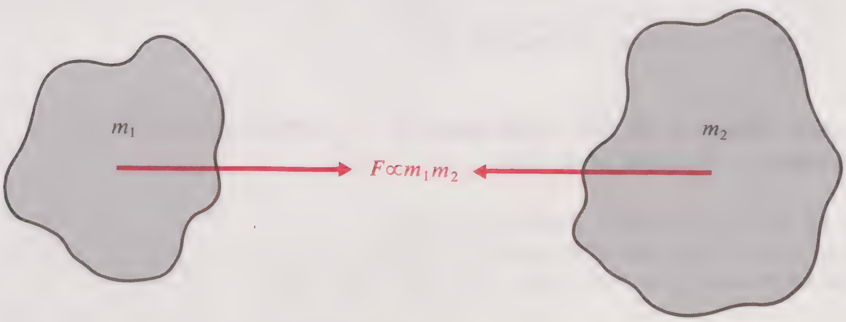


FIGURE 13 The gravitational force of attraction is proportional to the product of the masses of the attracted objects, $F \propto m_1 m_2$.

SAQ 14 Our solar system as viewed from a distant star can be fairly accurately described as consisting of a stationary Sun around which the planets move in approximately circular orbits. What does the lack of movement (small acceleration) of the Sun imply about its mass?

4.5 Newton's third law of motion

The conclusion that the force on the Moon is equal to the force on the Earth is a manifestation of an important principle which is true whenever two objects interact. *The forces they exert on each other are equal but act in opposite directions.* A simple example will illustrate what this means:

interaction (of objects)

> Why is it possible to stand still even though your weight (a force) is always acting to accelerate you downwards?

The problem is similar to the one you met earlier in SAQ 9 and the answer is the same: the floor pushes up exactly as hard as your weight pushes down. The forces balance, and, because there is no net force, there is no net acceleration. Now that there is Newton's general principle relating the forces between interacting objects to appeal to, this 'coincidence' is no longer so remarkable. You and the floor are interacting objects: your weight pushes *down* on the floor and so the floor pushes *up* with an equal but opposite force (Figure 14).

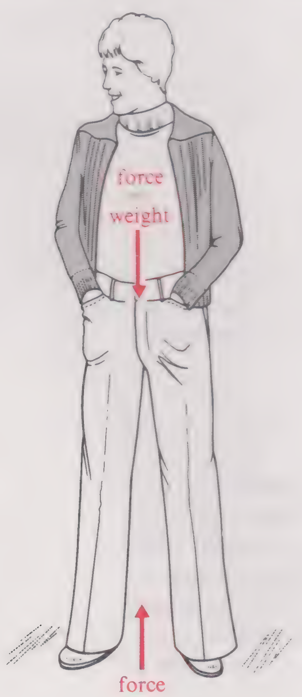


FIGURE 14 The weight of a standing person is *exactly* balanced by an upward force from the floor.

Although the general idea had been suggested earlier, it was Newton who realized that such a principle was essential in the development of his theory of motion. Typically he set to work and checked whether it was consistent with his observations on a number of systems. In each case the principle was upheld and with this confirmation Newton was able to formulate the principle as his third law of motion:

> **When two bodies interact, the force exerted by the first on the second is equal and opposite to the force exerted by the second on the first.**

Newton's third law

You have now been presented with all three of Newton's general statements (laws) about motion. In Section 5 you will verify his theory of gravitation; the combination of the four ideas is able to explain all Kepler's laws, and much more.

SAQ 15 Why does a gun recoil when it is fired?

SAQ 16 Why does the bullet accelerate more than the gun?

4.6 Objectives of Section 4

You should now be able to:

(a) Recall that the gravitational acceleration of an object is independent of its mass (Home Experiment on free-fall).

(b) Relate the mass of an object to its weight through Newton's second law (SAQ 13).

(c) Explain that the acceleration of the Sun is much less than that of the planets because of its much greater mass (SAQ 14).

(d) State Newton's third law of motion and use it to explain simple observations of interacting objects (SAQs 15 and 16).

5 Verifying the law of gravity

This Section carries the investigation of the gravitational force further forward through experiment and analysis. By the end you will have verified the dependence of the force on distance. At the moment you may not feel completely at home with the abstract ideas of the previous Sections. If so, don't worry; the use of these ideas in a more practical context may help you to get used to their meaning.

5.1.1 Acceleration due to gravity (Home Experiment)

An object accelerating towards the surface of the Earth is probably the most familiar manifestation of gravitational effects and so the first investigation is of this acceleration, to which we will give the symbol g_E. Of course g_E is the same at any one place for all objects (Section 4.2). The experiment is simple, but don't be tempted to skip it as your data will be used to introduce and illustrate the important subject of errors. Do the experiment now.

Home Experiment 1

Take the stop-watch from your Home Experiment Kit and find about a dozen small objects such as pebbles or dried peas. Hold a pebble out of an upper window*, release it, and time how long it takes to fall to the ground (Figure 15). You might find it helpful to drop the pebble into a bowl of water as this will help you to see the exact moment of impact. Now you will find that the time of fall for a typical two-storey house is rather short and that it is quite difficult to get a good measurement. Even if your reflexes are perfect, the stop-watch we have provided has divisions of 0.2s., an appreciable fraction of your total time. So practise your technique until you're satisfied that you are making as accurate a measurement as possible.

Then, take say five readings of the time of fall. Finally, use your own initiative** to find a way of measuring the height through which you have been dropping the pebbles.

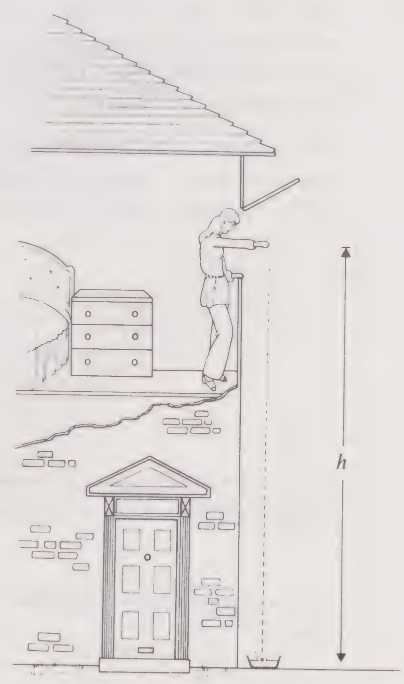


FIGURE 15 Home Experiment to measure g_E. The time t a pebble takes to fall a distance h is measured by a stopwatch.

* The bedroom window of a two-storey house. The accuracy of the experiment will improve for greater heights but, if you live in a bungalow, don't worry. Drop the pebble from the ceiling or use a stepladder outside. If you live in a taller building such as a block of flats, do be careful about dropping things out of the window. Perhaps it would be wise to get a collaborator to stand below and keep the coast clear.

** For instance, you could pay out a length of string from the window until it just reaches the ground. The length of string is then the distance of fall, and the error is probably only a few cm.

5.1.2 Analysis of pebble experiment

You should have five time readings and one height reading. Rather than carrying through the analysis for all the data, it is more convenient to reduce the time readings to one value—their average. So, add the five times together and divide by five. If you're unsure about why this operation should be the way to find the average time, take a look at the relevant section in *HED**. **HED**

How large is the possible error in this average time reading?

Even if you have performed the experiment perfectly, the design of the stopwatch allows an error of up to 0.1 s in each time-reading (i.e. 1.69 s is recorded as 1.6 s and 1.71 is recorded as 1.8 s). So the error in the average might be up to 0.1 s. But have you performed the experiment perfectly, or are there additional errors? We can estimate the random 'operator error' from the spread of readings and, to some extent, the taking of the average will have reduced this error. If it is random, you are just as likely to have got a high reading as a low one and these will tend to cancel each other out. A systematic error is trickier to find. For example, the readings may all agree but be 0.2 s too long because you were systematically slow in recording the moment of impact. There is no easy way out of this difficulty: only you can estimate how large your systematic error might be. Bear these points in mind, and if you want further guidance, read the discussion of errors in *HED* before you decide on the error appropriate to your average time. Estimate also the error in your measurement of the height. From your data record: **HED**

time taken to fall distance h = average time ± error

t = ... ±. . . . . . . seconds

and h = ... ± . . . . metres

It is often useful to use the term 'percentage error'. You should remember from Unit 2 that this is the ratio of the error to the reading, expressed as a percentage**. For this experiment the percentage error in the value of h should be much smaller than the percentage error in t, and so we will neglect the error in h in the analysis.

The next task is to find out how to calculate the acceleration g_E from the distance h and the time t. Remember that the acceleration is constant and that the pebble started with zero velocity. From the definition of acceleration,

$$g_E = \frac{\text{change in velocity}}{\text{time}} = \frac{\text{final velocity } v_t - 0}{\text{time } t} = \frac{v_t}{t} \qquad (23)$$

The final velocity can be expressed in terms of the acceleration and time by multiplying both sides of the equation by t,

$$v_t = g_E t \qquad (24)$$

The average velocity is half the final velocity (since the initial velocity was zero), that is:

$$v_{\text{average}} = \tfrac{1}{2} g_E t \qquad (25)$$

But the average velocity is also equal to the distance travelled, h, divided by time, that is:

$$v_{\text{average}} = h/t \qquad (26)$$

* The Open University (1979) S101 *The Handling of Experimental Data* (*HED*), The Open University Press.

** Suppose the reading is 20 and the error is ±1. The fractional error is 1/20. Expressed as a percentage, 1/20 is equivalent to 5 per cent. So the percentage error is 5 per cent. Percentages are discussed in Block 1 of *MAFS*. **MAFS 1**

Put these two expressions for the average velocity together and we have an equation which links g_E, h and t:

$$v_{\text{average}} = \tfrac{1}{2}g_E t = h/t \tag{27}$$

How should this equation be rewritten so as to obtain an expression for g_E?

The equation can be rearranged by multiplying both sides by 2 and dividing both sides by t. Now we have

MAFS 2

$$g_E = 2h/t^2 \tag{28}$$

This equation, which will be referred to several times later in the Unit, provides a means of calculating g_E from your data, but in doing so it is necessary to keep track of the uncertainties. So, using $g_E = 2h/t^2$, calculate g_E for both the average value of t and the values of t corresponding to the limits of your experimental uncertainty. These calculations provide you with a 'best value' for g_E and the magnitude of the likely error in this value:

$$g_E = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots \text{m s}^{-2}$$

Calculate the percentage error for your value of g_E and compare it with the percentage error for your average time reading.

This is an instructive calculation. To calculate g_E you squared the value of t. This should have *doubled* the percentage error*. This increased uncertainty may leave you rather disappointed with the outcome of the experiment: you perhaps have a possible error in your value for g_E of 40 per cent. Don't despair, the experiment is meant to illustrate the difficulties the scientists in the sixteenth and seventeenth centuries must have had with their much less sophisticated timing apparatus. As a bonus, in this Section, we have also introduced many of the ideas we shall use in the next Section. If you have had difficulty with this Section, Appendix 1 analyses a specimen set of data for the experiment.

SAQ 17 The volume V of a cube of side l is given by $V = l^3$. The length of a side is measured as (0.100 ± 0.002) m. What is the percentage error in l and in V?

5.2 Stroboscopic determination of g_E

How can we verify Newton's theory of gravitation? To do so, an accurate value for g_E is needed. Clearly, the previous simple experiment is inadequate. Rather than adopting a less direct method, we will keep the basic principle of the experiment the same and reduce the errors by using more accurate apparatus. Unfortunately, the principal item we will need, the stroboscope, cannot be sent to you in the Home Experiment Kit. Instead the 'raw results' of the experiment are reproduced in the text for you to analyse by a graphical method.

The method turns the previous Home Experiment on its head. Instead of measuring how long it takes for an object to fall a fixed distance, you will measure how far it falls in a fixed time, using a stroboscopically lit photograph.

stroboscope

A *stroboscope* is a device that provides short pulses of light at regularly spaced intervals of time. Therefore, if a scene lit only by a stroboscope is photographed with a long exposure time, the photograph appears as a superimposed sequence of images conveying the motion that has taken place. When the length of each flash is sufficiently short, the photograph is an unblurred record of the scene at each flash.

* This is one example of a general rule. If you measure a time as (100 ± 1) s, then the percentage error is 1 per cent. Square the time reading and the limits allowed by the errors are $(99)^2\,\text{s}^2 \approx (10\,000 - 200)\,\text{s}^2$ and $(101^2)\,\text{s}^2 \approx (10\,000 + 200)\,\text{s}^2$. So the square of the time reading is $(10\,000 \pm 200)\,\text{s}^2$, an error of 2 per cent. Raise the time reading to the third power and you have $(10^6 \pm 3 \times 10^4)\,\text{s}^3$, a 3 per cent error. And so on. Raise a reading to the nth power and the percentage error is multiplied by n. You will find more about errors in the powers of quantities in *HED*.

HED

We have taken (Figure 16) a picture of two balls falling through the air between two scales. The release of the balls coincides with the first flash of a stroboscope, and the time interval between flashes is 40 milliseconds. The successive images of the balls show their positions after 40 ms, 80 ms, 120 ms, etc. Can you see how the photograph provides confirmation of a conclusion reached earlier in the Unit—that the acceleration due to gravity is independent of mass?

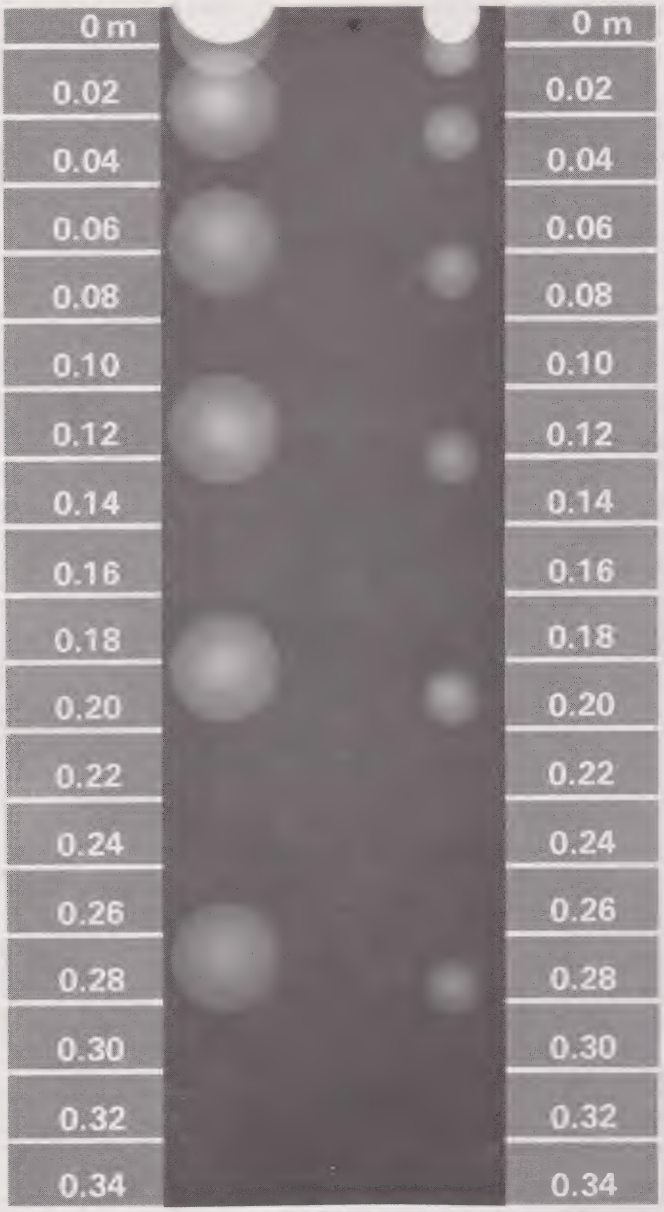


FIGURE 16 A stroboscopically lit photograph of two balls falling through the air. You have possibly noticed that the positions of the two balls are, particularly for the final flash, very slightly different. The small discrepancy arises from the shortcomings of the magnetically based method of releasing them. In our analysis of the photograph we have taken the average position of the two balls for each flash.

SAQ 18 How far have the balls travelled in the first 0.16 s from their release?

The picture contains all the information you would need to plot a graph of distance travelled h, against time of fall, t. We have done so in Figure 17. As we will shortly be asking you to plot a related graph using the same data, Figure 17 only shows the form of the curve—there is no scale on the graph.

Why does the slope of the curve in Figure 17 increase with time?

An increasing slope indicates that the distance travelled in a given length of time is increasing, that is, the velocity is increasing. The change of slope reflects the acceleration of the balls.

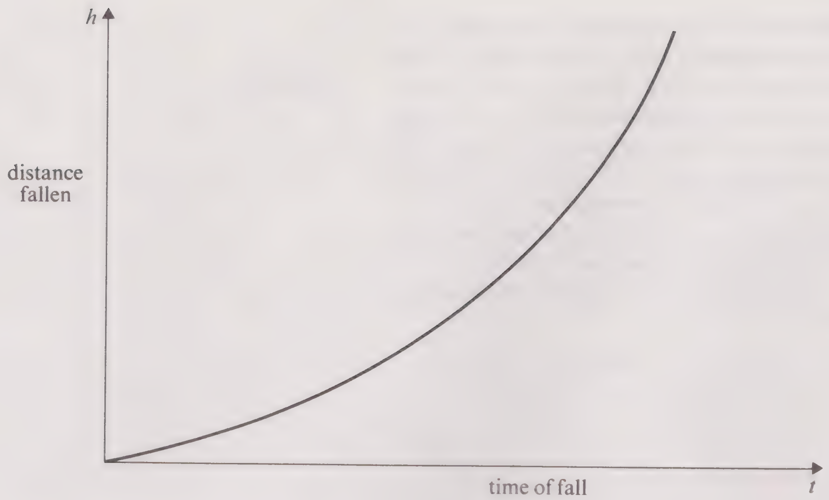


FIGURE 17 If the 'data' from the stroboscope experiment were plotted on a graph with axes, h and t, the points would lie on a curve of this shape.

Unfortunately it is not easy to extract a value for g_E from this graph. Instead the data must be replotted in a more useful format.

An analogous example will show how this can be done. The area of a circle of radius r is given by $S = \pi r^2$. A graph of S plotted against r has the form shown in Figure 18. As in the graph of h against t, the line, which in this case represents the equation $S = \pi r^2$, is curved. But look at Figure 19, in which S is plotted against r^2. It is a straight line. The point to notice is that the quantities represented by the axes in Figure 19 are proportional to each other. S is proportional to r^2. The general rule is that a graph in which proportional quantities are plotted against each other is always a straight line.

MAFS 4

MAFS 3

SAQ 19 The volume of a sphere is proportional to the cube of the radius, that is, $V = (4/3)\pi r^3$. Which choice of quantities to plot will allow this relationship to be represented as a straight line?

The same technique for 'straightening out' a curved graph can be used to analyse the data from our falling-ball experiment. The data should obey equation 28:

$$g_E = 2h/t^2$$

This can be rewritten

$$h = \tfrac{1}{2}g_E t^2 \tag{29}$$

SAQ 20 What quantities plotted against each other will produce a straight-line graph representing this equation?

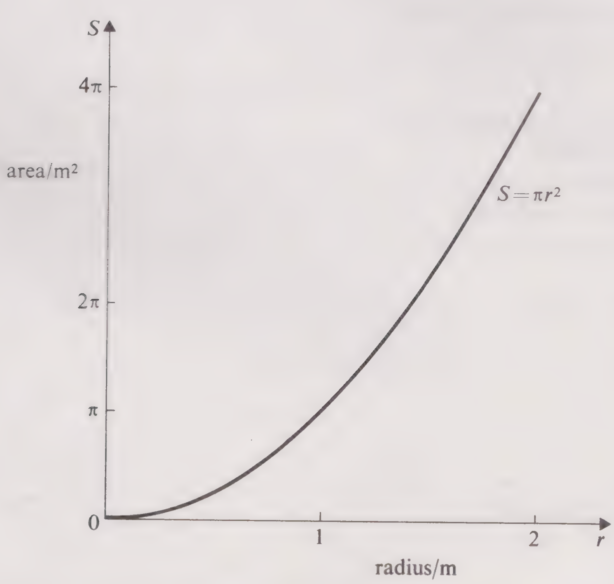


FIGURE 18 With axes S and r, the equation $S = \pi r^2$ is represented by a curve of the same shape as that in Figure 17.

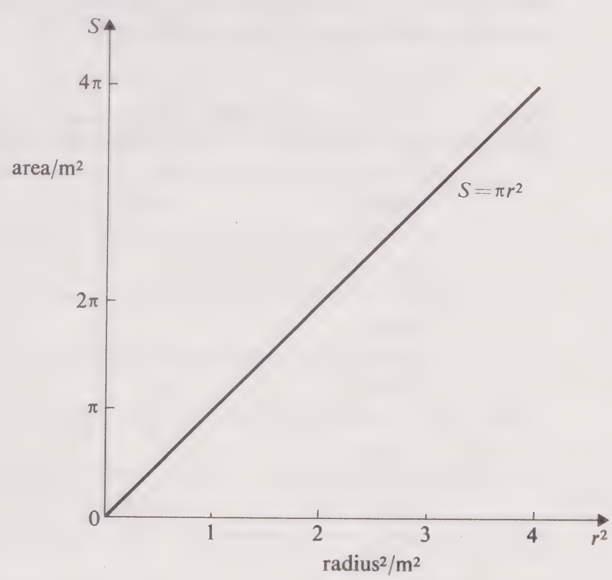


FIGURE 19 With axes S and r^2, the equation $S = \pi r^2$ is represented by a straight line; S is proportional to r^2.

We have already emphasized the importance of estimating the uncertainty of experimental data. In plotting graphs the uncertainty can be represented by the use of *error bars*. For example the answer to SAQ 18 gave the distance travelled by the balls in 0.16 s as 0.128 ± 0.003 m. The flashing of the stroboscope is accurately controlled, and therefore we can safely neglect any error in the value of t, but the correct value for h corresponding to $t = 0.16$ s ($t^2 = 0.0256\,s^2$) might lie anywhere between 0.125 m and 0.131 m. In recognition of this uncertainty, the 'point' is plotted on Figure 20 as a line (an *error bar*) covering the possible values of h. You should include error bars on all your plotted points. Now fill in Table 1 from the photographed data (Figure 16) and plot the values of h against t^2 on Figure 20.

error bars

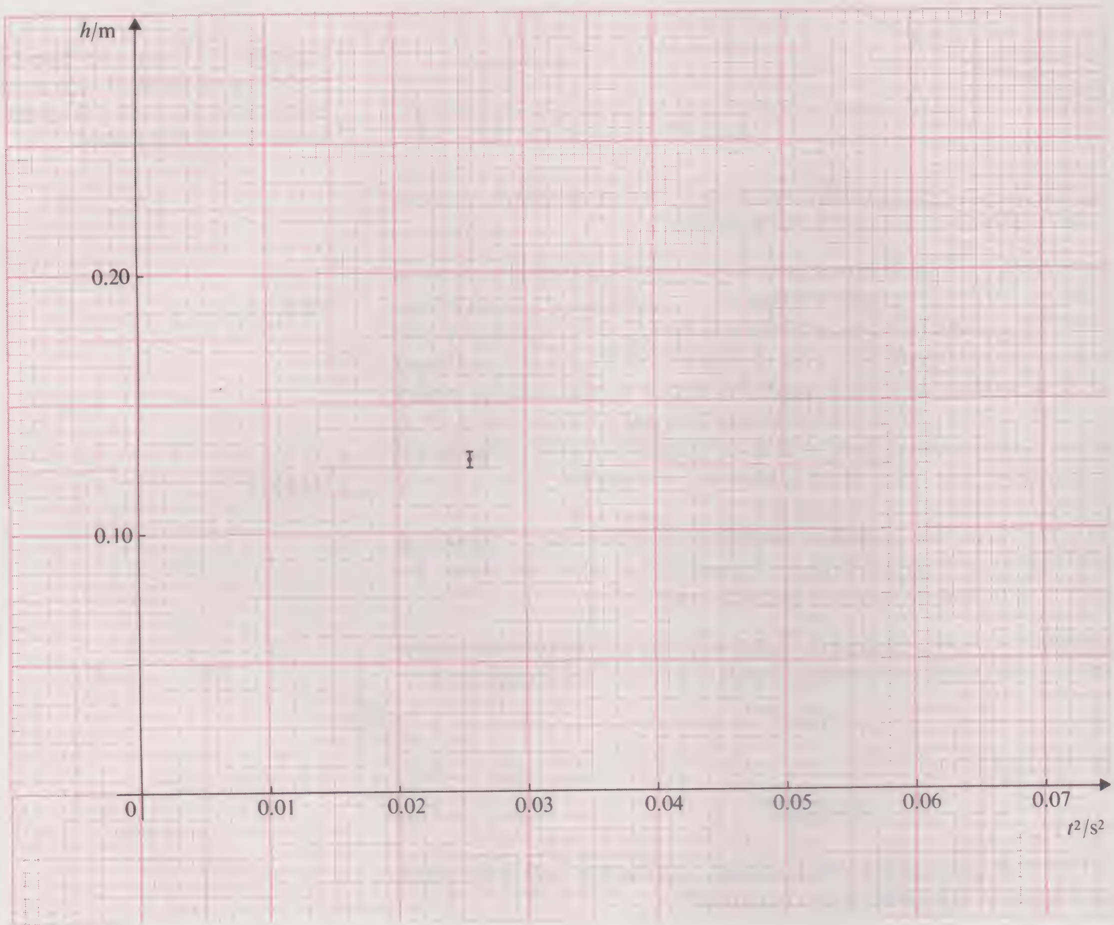


FIGURE 20 On this graph you should plot, with the appropriate error bars, the data contained in the stroboscope picture (Figure 16).

TABLE 1 The relationship between distance travelled by the two falling balls shown in Figure 16 and the time for which they have been falling.

time/s	time^2/s^2	distance travelled/m
0		
0.04		
0.08		
0.12		
0.16	0.0256	0.128 ± 0.003
0.20		
0.24		

Assuming you have made no mistakes, your graph should have turned out to consist of a set of error bars, through all of which a straight line could be drawn. Don't draw the line just yet. If your result isn't as expected, check your working and, if desperate, refer to Appendix 2 for our analysis.

What does the slope of the line represent?

Until now we have talked rather loosely about the 'slope' of a line or curve, using its colloquial meaning, but the time has come to be precise. A particularly useful measure of 'slope' is the *gradient*, the change in the value of the quantity on the vertical axis divided by the change in the value of the quantity on the horizontal axis between two points on the curve. That is rather a mouthful and you will probably appreciate an example illustrating this definition.

gradient

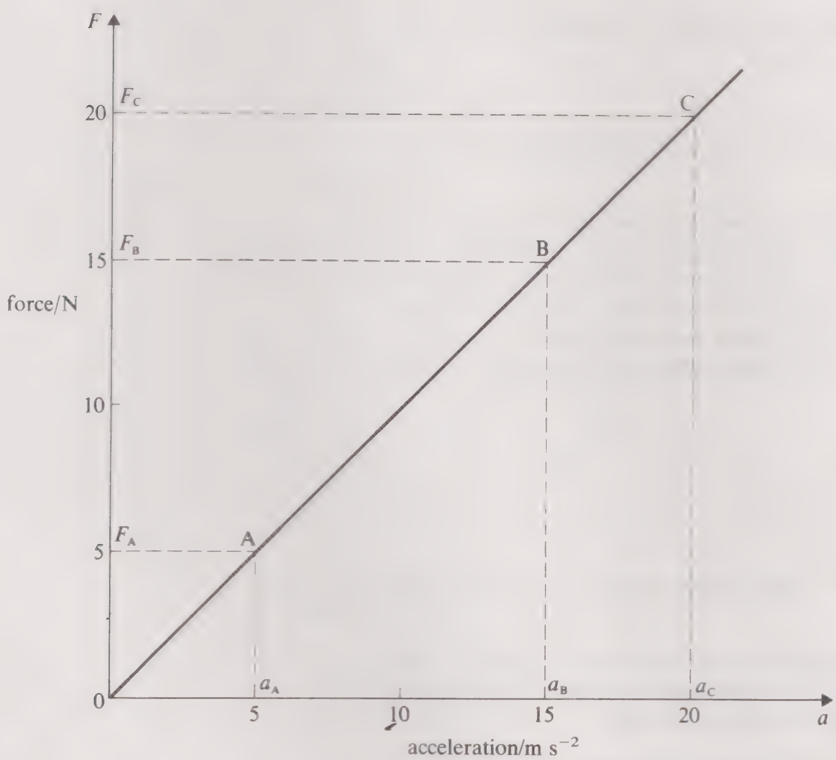


FIGURE 21 The force F on a 1 kg mass is plotted against the acceleration a it produces.

Look at Figure 21, which shows a graph of the force F on an object of mass 1 kg, plotted against the acceleration a that it produces. The gradient is the change in force $(F_B - F_A)$ divided by the change in the acceleration $(a_B - a_A)$ between the points A and B, that is:

$$\text{gradient} = \frac{F_B - F_A}{a_B - a_A}$$

From the graph,

$$F_B - F_A = (15 - 5)\,\text{N} = 10\,\text{N}$$

and

$$a_B - a_A = (15 - 5)\,\text{m}\,\text{s}^{-2} = 10\,\text{m}\,\text{s}^{-2}$$

And so, the gradient of the line is:

$$\text{gradient} = 10\,\text{N}/10\,\text{m}\,\text{s}^{-2} = 1\,\text{N}/\text{m}\,\text{s}^{-2}$$

The units associated with the gradient are rather complicated but they can easily be simplified. The dimensions of force/acceleration are (mass × length × time^{-2})/(length × time^{-2}) = mass.

So the gradient of the line between points A and B is one unit of mass = 1 kg.

SAQ 21 What is the gradient of the line between points A and C?

In fact you get the same value for the gradient irrespective of the points between which you choose to calculate the gradient: the gradient of a straight line is

constant*. For our example the line has a gradient of 1 kg. Now this is not a coincidence, for the straight line is a graphical representation of Newton's second law, $F = ma$, and the gradient is the constant m (which we chose to be 1 kg) linking the quantities on the axes, F and a.

In general, if two quantities, y and x, are linked by the equation $y = kx$ the gradient of the graph in which y is plotted against x is k. **MAFS 3**

SAQ 22 From the stroboscope picture you have plotted values of h against t^2. These quantities are linked by the equation $h = \frac{1}{2}g_E t^2$. How is the gradient of your line of error bars related to g_E?

Now draw in on Figure 20 the line that goes most centrally through the error bars of all individual data points and the steepest and shallowest lines that are also consistent with the error bars**. Calculate the gradient of each of these lines. **HED**

From the answer to SAQ 22 you should now be able to calculate a best value for g_E and give an uncertainty in that value.

$$g_E = \ldots\ldots\ldots \pm \ldots\ldots\ldots \mathrm{m\,s^{-2}}$$

If you were unable to calculate a value for g_E, or to estimate the uncertainties, then you should refer to Appendix 3.

Clearly the stroboscopic technique leads to a much more accurate determination of g_E, probably within a few per cent of the accepted value, $g_E = 9.81\,\mathrm{m\,s^{-2}}$. Of course, there are even more precise methods, and these have produced the interesting result already mentioned in SAQ 13: they have shown that there is a small geographical variation in g_E. At the North Pole g_E is $9.833\,\mathrm{m\,s^{-2}}$ whereas at the Equator g_E is $9.781\,\mathrm{m\,s^{-2}}$, a 0.5 per cent difference. The reason for this variation will be discussed in Unit 4.

5.3 The motion of the Moon

The value of g_E is a quantitative description of the magnitude of the force of gravity felt on the Earth's surface.

Can the same exercise be carried through for the Moon and the size of the force of gravitational attraction between the Earth and Moon discovered? It can, but to do so a closer look at the Moon's motion is needed.

The motion of the Moon is characterized for present purposes by two quantities, the radius and period of its orbit. You found the first of these in Unit 2, but remember that the original method assumed incorrectly that the radius of the Earth's shadow on the Moon was equal to the actual radius of the Earth. You should have corrected your value for the distance to the Moon (d_M) after watching the TV programme associated with Unit 2 (TV 02), but, in spite of this, your value for d_M may still be too much in error for the analysis of this Section. Instead, use the value found from laser ranging:

$$\text{mean radius of Moon's orbit, } d_M = 3.84 \times 10^8\,\mathrm{m}$$

The period of the Moon's orbit is approximately 28 days, the lunar month:

$$\text{period of Moon's orbit} = 28 \text{ days}$$

These two values are approximations but the errors involved in using them are small compared with the errors that will be introduced later in the analysis.

Our goal is to find the *orbital acceleration* of the Moon due to the gravitational force of attraction of the Earth. But how is it possible to do this: how is it possible to work out the acceleration of an object like the Moon which is in a circular orbit? **orbital acceleration**

* In practice, you should choose points that are well separated on the line. This will ensure that any errors in identifying the points do not lead to a large error in the calculation of the gradient.

** The use of error bars on graphs is discussed more fully in *HED*.

Look at Figure 22. If a stone is thrown horizontally it follows a curved path (path A) before *falling* back to Earth. Shoot a rifle bullet at the same angle and it too will eventually *fall* back to Earth (path B), but its range is much greater. Now suppose it were possible to continue to fire projectiles horizontally but with even greater velocity. As the velocity increased, the range (path C) would increase (we are ignoring the complications of air friction) until there would come a time when the projectile would never reach the ground. As fast as it *fell* towards the ground (path D), the surface of the Earth would be curving away from it. The projectile would be in orbit. *At all times it would be accelerating back towards the centre of the Earth* but the continuous fall would only be able to compensate for the curvature of the Earth. Orbiting can be considered as a process of continuous falling and it is this idea which is the basis of a method to find the orbital acceleration of the Moon. Incidentally the method is the same as that used by Newton.

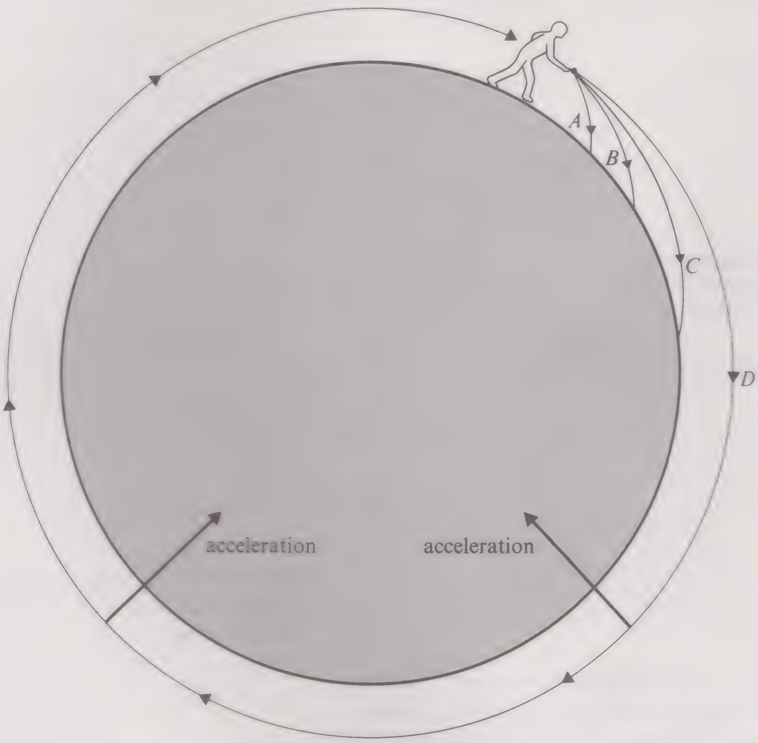


FIGURE 22 A projectile thrown with the correct horizontal speed would go into orbit if we could somehow remove the air which stops it.

Read the instructions through completely and then find the orbital acceleration of the Moon*.

1 Construct a scale model of the Earth–Moon system by drawing a circle of radius 10 cm to represent the Moon's orbital path. (Scale: 10 cm is equivalent to d_M.)

2 The Moon goes completely round the Earth in 28 days (360° in 28 days). How large is the angle through which it moves in one day? Draw this angle on your diagram, using a protractor marked in degrees. This construction is shown *schematically* in Figure 23, p. 34. For convenience the Moon is assumed to be at the top of the diagram, position Q, at the start of the day.

3 At the start of the day, the Moon is moving 'horizontally' on the page. During the day it falls a distance h to point P. Measure the distance h on your scale model, not forgetting to estimate the uncertainty in your measurement.

$h =$ .... $\pm$ .... mm

4 Convert h to a real distance for the Earth–Moon system by using the scaling factor for your diagram:

Moon 'falls' in one day .... $\pm$ .... m

5 Express one day in seconds

$t =$ one day $=$ .... seconds

* If you are unable to do this, refer to Appendix 4.

6 Your h and t are analogous to the data that you used to estimate the acceleration of a falling object at the Earth's surface. The equation $h = \frac{1}{2}a_M t^2$ relating the distance fallen to the time taken is again valid. (When you last met this equation, we were dealing with an object falling at the surface of the Earth and so the acceleration was g_E, not a_M.) Deduce your value for the orbital acceleration of the Moon:

$$a_M = 2h/t^2$$

$$a_M = \dots\dots\dots \pm \dots\dots\dots \mathrm{m\,s^{-2}}$$

Assuming all your measurements and calculations are correct, you will have found that the acceleration of the Moon a_M is much smaller than the acceleration of an object at the Earth's surface, g_E. In effect, your measurement has shown a very significant feature of the gravitational force of attraction between two bodies: it gets progressively weaker as the distance between them increases*.

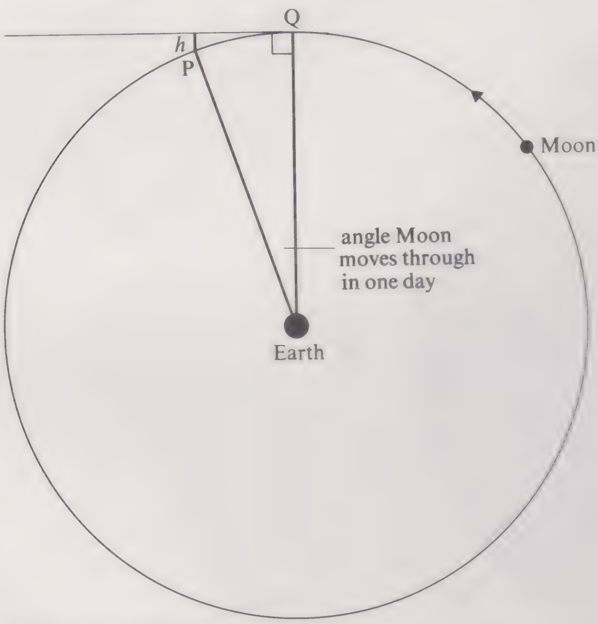


FIGURE 23 Construction to estimate the distance the Moon 'falls' in one day. The diagram is *not* to scale.

Newton realized that observations similar to yours could provide a way of checking any proposed mathematical relationship between the gravitational force and the distance between the attracted objects, but there was a sizeable problem to be overcome. How should he calculate the gravitational force acting on a small object (the falling ball) on the surface of an immensely larger one (the Earth)? After all, the ball is attracted to all the bits of the Earth, both near and far. What is the net force? Now this problem wasn't trivial and Newton took 20 years to solve it. He was forced to invent a totally new branch of mathematics which we now know as integral calculus. Fortunately, the conclusion he came to simplified the problem considerably. For the universal law of gravitation he was to propose, the entire mass of the Earth could be assumed to be concentrated at its centre.

With this problem behind him, Newton was able to check the expression that he suspected was appropriate for the force–distance relationship. We have already established that the gravitational force between two objects is proportional to the product of their masses. Newton believed that the force was in addition inversely proportional** to the square of the distance between these masses.

* Remember that the acceleration of an object due to gravitational forces does not depend on its mass. The enormous difference in the masses of your two test objects, the Moon and the ball, does not affect the conclusion of the experiment.

** The quantities x and y are inversely proportional if

$$x = \text{constant} \times \left(\frac{1}{y}\right)$$

For example, the volume of a fixed mass of gas at a given temperature is inversely proportional to the pressure. If the pressure increases, the volume decreases and vice versa. $V \propto 1/P$. Proportionality is discussed in Block 1 of *MAFS*.

MAFS 1

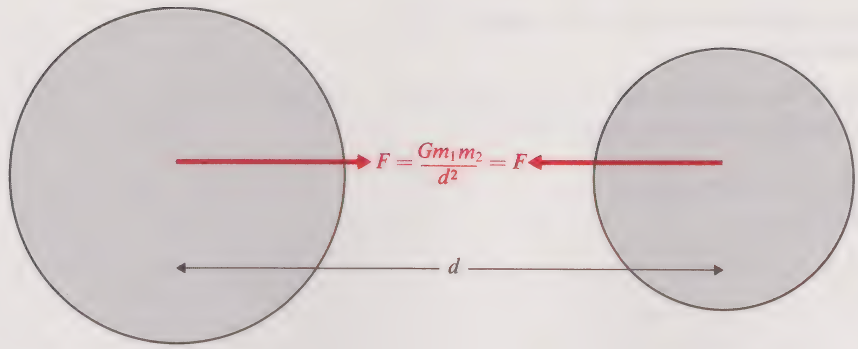


FIGURE 24 The attractive force F between two objects of mass m_1 and m_2 which are a distance d apart is given by $F = Gm_1m_2/d^2$.

For the two objects in Figure 24, these two statements can be combined in the equation

$$F = G\frac{m_1m_2}{d^2} \tag{30}$$

Newton's theory of gravitation

where G is a *universal constant*, that is, *it has the same value everywhere, for all masses*. How Newton tested this expression is revealed in his *Principia*:

universal constant

> I began to think of gravity extending to the Orb of the Moon . . . . I deduced that the forces which keep the Planets in their Orbs must be reciprocally as the squares of their distances from the centres about which they revolve: and thereby compared the force requisite to keep the Moon in her Orb with the force of gravity at the surface of the Earth, and found them answer pretty nearly.

You can use the results of the experiments in this Unit to check if Newton was correct when he 'found them answer pretty nearly'. If the force of attraction falls off as the inverse square of the distance, so also must the acceleration. Your first test object, the ball, had an acceleration of g_E at a distance R_E (use the correct value of 6.38×10^6 m) from the Earth's centre. The second, the Moon, had an orbital acceleration a_M at a distance d_M from the Earth's centre. If Newton was right,

$$\frac{\text{acceleration of ball-bearing}}{\text{acceleration of moon}} = \left(\frac{\text{distance to Moon}}{\text{radius of Earth}}\right)^2$$

that is,

$$\frac{g_E}{a_M} = \left(\frac{d_M}{R_E}\right)^2 \tag{31}$$

The right-hand side of this equation can be calculated precisely as there is negligible error in the data we have given, but, in calculating, from your own data, a value for the left-hand side g_E/a_M, you should make allowance for and combine the separate uncertainties in g_E and a_M. An example will show how this can be done.

Suppose that two quantities x and y are measured as

$$x = 5 \pm 1 \quad \text{(20 per cent error)}$$

and

$$y = 10 \pm 1 \quad \text{(10 per cent error)}$$

Notice that the percentage errors in x and y are comparable and so it is *not* safe to neglect the error in y in calculating the error in x/y. In Unit 2 the combined error was determined by taking the extreme values of each quantity; by this method the value of the ratio could lie anywhere between $x/y = (5 + 1)/(10 - 1) = 6/9 \approx 0.67$ and $x/y = (5 - 1)/(10 + 1) = 4/11 \approx 0.36$. So, approximating the errors, the ratio could be quoted as

$$x/y = 0.50 \pm 0.15 \quad \text{(30 per cent error)}$$

The separate 20 and 10 per cent errors have been combined to produce a 30 per cent error in the ratio.

> Is this result a fair representation of the uncertainty in the data?

In fact, this method is unduly pessimistic. It assumes that, in calculating x/y, we have chosen values for x and y which are *both* at the limits of their respective

errors. But these measurements are independent and so there is no reason why there should be such an unhappy coincidence.

How then are the errors to be combined? As is pointed out in *HED*, statistical theory provides the answer: For our example, the general result reduces to:

HED

$$\begin{aligned}\text{percentage error in } x/y &= \sqrt{(\%\text{ error in } x)^2 + (\%\text{ error in } y)^2} \qquad (32)\\ &= \sqrt{(20)^2 + (10)^2}\text{ per cent}\\ &= \sqrt{500}\text{ per cent}\\ &\approx 22\text{ per cent}\end{aligned}$$

But 22 per cent of 0.50 = 0.11

So $\qquad x/y = 0.50 \pm 0.11$

The result is quite reasonable: the combined percentage error is greater than either one of the individual percentage errors but is not as large as their sum.

The same statistical technique can be used to find the error in the *product* of two independent quantities, x and y. For both the product xy and the ratio x/y, the percentage error is given by equation 32.

In fact, it is only necessary to combine the errors when the percentage errors are about the same. As a useful rule of thumb, if one of the quantities has a percentage error three or more times that of the other quantity, only the larger error need be considered.

Now you should be able to calculate a value for g_E/a_M from your own data:

$$g_E/a_M = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

$$(d_M/R_E)^2 = \ldots\ldots\ldots\ldots$$

Are your values of g_E/a_M and $(d_M/R_E)^2$ the same within the uncertainties estimated for them? They should be*. If not, refer to Appendix 5 where the analysis has been carried through for our data.

We hope that you have verified that Newton's law of gravitation is consistent with your data. You have not proved it to be true; the agreement could be fortuitous, but in fact far more precise measurements on a much greater variety of systems also give data consistent with Newton's law. It is therefore accepted as a true description of the gravitational force between any two masses separated by any distance.

SAQ 23 Calculate the area of a rectangle, and the possible error in this value, if two adjacent sides a and b are measured as 2.0 ± 0.2 cm and 1.0 ± 0.1 cm.

5.4 Objectives of Section 5

After reading this Section *and* performing the associated Home Experiments and Analysis, you should be able to:

(a) Describe the light output of a stroboscope as consisting of a regular series of short pulses of light (SAQ 18).

(b) Describe an experiment to measure the acceleration due to gravity at the surface of the Earth (Home Experiment).

(c) Explain how the orbital motion of the planets can be understood in terms of gravitational forces by invoking a picture of continuous falling towards the centre of the orbit (Home Analysis).

(d) Estimate the gravitational acceleration of the moon (caused by the pull of the Earth), given the radius and period of the moon's orbit (Home Analysis).

* Newton had some difficulty doing this. At first he showed that they *weren't* equal. Only after a lot of trouble did he find that the ratio, Moon's orbital radius/Earth's radius, which he used, was wrong.

(e) State the mathematical form of Newton's theory of gravitation (that is, $F = Gm_1m_2/d^2$) and describe one method of verifying it (Home Analysis).

In addition, you should be able to:

(f) Calculate the percentage error in a reading given the reading and the associated error, and calculate the percentage error in the nth power of the reading (SAQ 17).

(g) Recognize that expressions of the form $y = kx$, $y = kx^2$, or generally $y^a = kx^b$ can be represented on a graph as a straight line by an appropriate choice of axes (SAQs 19 and 20).

(h) Define the gradient of a line and relate it to the constant k linking the quantities on the axes (SAQs 21 and 22).

(i) Represent uncertainties in data by error bars on graphs (Home Analysis).

(j) Combine the errors in two readings to calculate the error in their product or ratio (SAQ 23).

6 The composition of the Earth and Moon

In the next four Units you will come down from the heavens to learn something about the structure of our own planet. The scientific methods introduced in the first three Units will help you. But before you start your study of earthquakes, magnetic fields and plate tectonics, it is interesting to see how even the subject matter of this Unit can reveal a little of the structure of the Earth.

6.1 The mass and density of the Earth

In the last Section you verified the mass and distance dependence of the gravitational force (that is, $F = Gm_1m_2/d^2$), but to calculate actual forces or accelerations, rather than ratios, the value of the constant G is needed. Now you might think that finding G only requires a simple experiment—place two bodies of known mass a known distance apart and measure the force of attraction—but in fact it isn't that easy. The gravitational force of attraction is very weak between everyday objects*. You don't find yourself attracted to every girder or wall or house you come near to. It is only because the Earth is so massive that its gravitational attraction is so apparent.

> **SAQ 24** In an experiment to measure G the force of attraction between two lead balls of mass 1 kg whose centres are 10 cm apart is measured to be 6.67×10^{-9} N. Calculate G.

Appropriately, one newton is about the weight of an apple. So, the force measured in the G-determination experiment, 6.67×10^{-9} N, is extremely small!

Now it turns out that such forces can be measured in laboratories but it requires great ingenuity and care. It wasn't until 1798 that Henry Cavendish carried out the first successful laboratory experiment to measure G (Figure 25). Even today G is only known to an accuracy of about one part in 10^4, far less precisely than most of the important physical constants you will meet later in the Course.

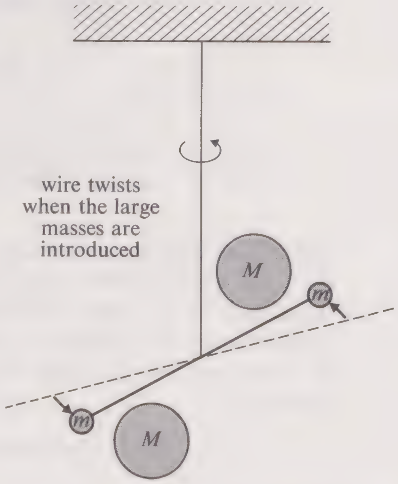


FIGURE 25 A schematic diagram of the apparatus Henry Cavendish used in the first laboratory determination of G. He inherited the apparatus from the Rev. John Michell, who had devised the method many years earlier. Fortuitously, Cavendish found a value for G which differed by only one per cent from the accepted value, even though he himself estimated that the uncertainty in the measurement was up to seven per cent.

Once G is known, it is relatively straightforward to find the mass of the Earth from the observed gravitational force on a test object. You should remember from earlier in the Unit that in computing the force on an object at the surface of the Earth, we could assume that the entire mass of the Earth was at its centre. So, the gravitational force of attraction between a ball of mass m and the Earth of mass M_E is, from Newton's law:

$$F = \frac{GM_Em}{R_E^2} \qquad (33)$$

* It is only of the order of 1 N between two oil-tankers moored side by side!

But this force of attraction acting on the ball is simply its weight, mg_E,

Therefore
$$mg_E = \frac{GM_E m}{R_E^2} \tag{34}$$

The equation can be simplified by dividing both sides by m, that is:

$$g_E = \frac{GM_E}{R_E^2} \tag{35}$$

The mass of the Earth can be expressed in terms of the other quantities by rearranging the equation. Multiplying both sides by R_E^2/G gives

$$g_E \frac{R_E^2}{G} = \frac{GM_E}{R_E^2} \times \frac{R_E^2}{G}$$

Therefore
$$M_E = \frac{g_E R_E^2}{G} \tag{36}$$

Now you can use your value for g_E and the accepted values for G ($6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$) and R_E to find the mass of the Earth. Don't forget to estimate the errors.

mass of Earth = ... ± .... kg

Our calculation of M_E can be found in Appendix 6. Of itself this result doesn't give much information about the structure of the Earth. More analysis is needed. What is the volume of the Earth? Is the mass what we would expect for a planet of this size?

The Earth is roughly spherical and a sphere has a volume $V = (4/3)\pi r^3$ where r is the radius. So it is possible to work out the total volume V_E as well as the mass M_E of the Earth.

Would dividing the total mass of the Earth by its total volume give the mass of unit volume of the Earth?

At first sight the answer to this question is obvious, yes, but that takes no account of any variation in the composition of the Earth. The masses of identical small volumes of the Earth might well vary from place to place. The quantity which describes this variation is the mass per unit volume or the *density*, which is usually given the symbol ρ (the Greek letter 'rho').

density

$$\text{density } \rho = \frac{\text{mass}}{\text{volume}} \tag{37}$$

SAQ 25 $10\,\mathrm{cm^3}$ of a hardwood called lignum vitae ('wood of life') has a mass of 13 grams. What is its density (in units of $\mathrm{g\,cm^{-3}}$)?

Density can be expressed in units of $\mathrm{g\,cm^{-3}}$ or $\mathrm{kg\,m^{-3}}$. As we are talking about the Earth we shall use the second of these in the rest of this Section. Although the density of the Earth can and does vary from place to place it is possible to calculate an average density, that is, total mass/total volume. Earlier we derived an expression for the total mass, $M_E = g_E R_E^2/G$. So the average density is given by

$$\rho\,(\text{average}) = M_E/V_E \tag{38}$$
$$= \frac{g_E R_E^2}{G} \times \frac{1}{(4/3)\pi R_E^3}$$

The right-hand side can be simplified by dividing above and below the line by R_E^2:

$$\rho\,(\text{average}) = \frac{g_E}{(4/3)G\pi R_E}$$

and therefore
$$\rho\,(\text{average}) = \frac{3g_E}{4G\pi R_E} \tag{39}$$

Use your value for g_E and the accepted values for G and R_E to find the average density of the Earth:

average density of Earth = ... ± .... $\mathrm{kg\,m^{-3}}$

You should have found a value somewhere near $5\,500\,\mathrm{kg\,m^{-3}}$. If you did not, refer to the calculation in Appendix 6.

> The rocks on the surface of the Earth have an average density of about $2\,600\,\mathrm{kg\,m^{-3}}$. Is the centre of the Earth the same as its surface?

To reconcile the average and surface densities, the material inside the Earth must be very different from that at the surface. The density must be much greater. You will learn more about this in Unit 4.

It is interesting that Newton *guessed* the average density of the Earth and with inspiration and luck was right. He reasoned that none of the solid ground could be less dense than water and that the Earth's core must be denser than its surface layer (or the centre of the Earth would flow to the surface). His guess was that the average density of the Earth was between 5 and 6 times the density of water ($\rho_{\mathrm{water}} = 1\,000\,\mathrm{kg\,m^{-3}}$)!

6.2 What might the Moon be made of?

Green cheese? Well probably not, but one way of checking is by measuring its average density, and thereby inferring something about its composition.

> Can we deduce the mass or density of the Moon from its acceleration towards the Earth?

Throughout the Unit we have stressed that the acceleration of an object due to gravitational forces is independent of its mass. Therefore, the *orbital acceleration* of the Moon cannot give a clue to *its own mass or density*. Instead, a method must be devised of measuring the Moon's effect on *another* body. Until the late 1960s such measurements were indirect. For example, the mass of the Moon could be deduced from the small changes it caused in the Earth's orbit round the Sun.

In 1969 one of the American astronauts on the Moon provided anybody who could watch him on television and use a stopwatch with an opportunity to measure the density of the Moon. In a re-creation of Newton's famous experiment, he demonstrated that on the airless surface of the Moon a feather and a hammer fall to the ground at the same time (Figure 26). You have already performed an

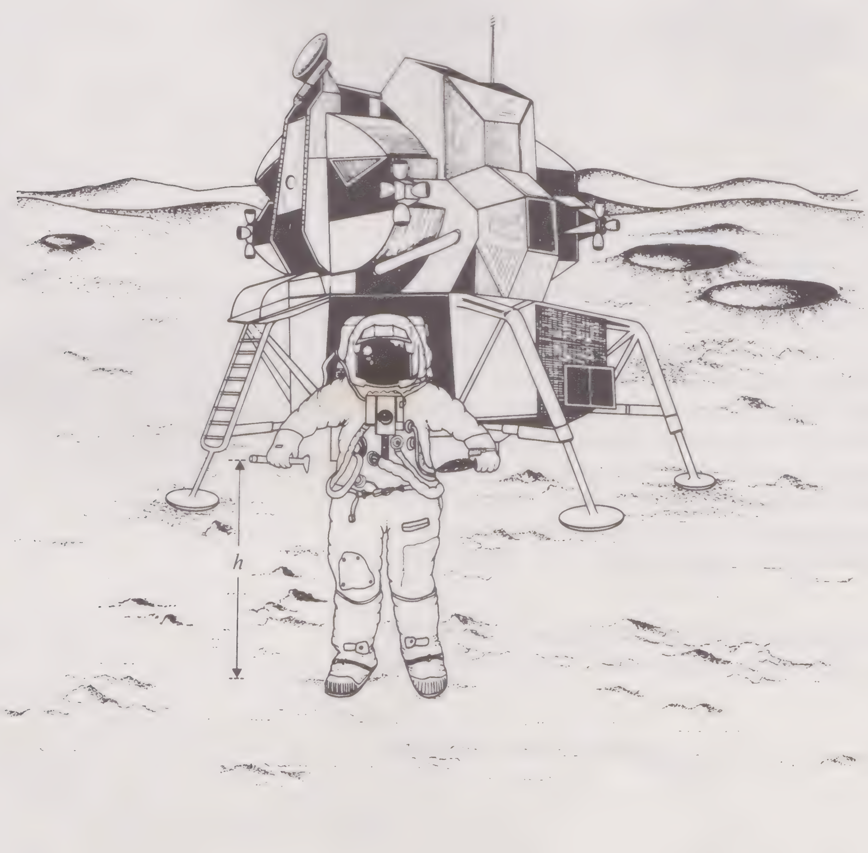


FIGURE 26 An astronaut performing the hammer and feather experiment on the surface of the Moon. As in the recording broadcast in TV 03, he is standing in front of the landing module.

earthbound experiment of this type by dropping the pebble out of an upper window. From the distance h travelled in a time t you derived the acceleration g_E. In turn, this allowed you to calculate the average density of the Earth. Now you can do the same for the Moon and in doing so, revise much of the content of this Unit.

A recording of the hammer and feather experiment is shown during TV 03. Unfortunately the quality of the pictures is rather poor, but nevertheless you should be able to estimate the height h from which the hammer was dropped. During the programme the time of fall t is measured: our value can be found in Appendix 7. Of course you may prefer to do the measurement yourself; if so, have your stopwatch ready.

$h =$ .... $\pm$ .... metres

$t =$ .... $\pm$ .... seconds

The analysis is now exactly analogous to that used in Section 6.1 to find the average density of the Earth. Use the relationship $g_M = 2h/t^2$ to calculate g_M, the acceleration of a falling object near the surface of the Moon.

$g_M =$ .... $\pm$ .... $\mathrm{m\,s^{-2}}$

Using the accepted value for the radius of the Moon ($R_M = 1.74 \times 10^6\,\mathrm{m}$) and the equation $\rho\,(\text{average}) = 3g_M/4G\pi R_M$, calculate the average density of the Moon.

average density of the Moon = ... $\pm$ .... $\mathrm{kg\,m^{-3}}$.

With luck you will have found a value for the average density of the Moon that is significantly lower than that of the Earth, but the large errors in the measurement of g_M may obscure this conclusion. The accepted value* is $3\,360\,\mathrm{kg\,m^{-3}}$, a figure not greatly different from the average density of the surface layers of the Earth. The Moon obviously isn't made of green cheese (which has a density of about $1\,100\,\mathrm{kg\,m^{-3}}$), and it isn't just a smaller copy of the Earth.

6.3 Objectives of Section 6

After reading this Section you should be able to:

(a) Calculate one of the five quantities F, G, m_1, m_2 or d in the equation $F = Gm_1m_2/d^2$ given the other four (SAQ 24).

(b) Define and use the concept of density (SAQ 25).

(c) Derive the mass of the Earth or Moon from measurements of their radii and the gravitational acceleration at their surfaces (Home Analysis).

7 Concluding remarks

In this Unit, we have discussed the ideas and the development of Newton's three laws of motion and his theory of gravitation. Newton, of course, didn't simply stop after enunciating these principles: he had to persuade his fellow scientists of their power and their truth. In his *Principia* he went on to apply his ideas to a whole host of observations: he showed that Kepler's laws are a consequence of the form of the gravitational force, he explained why the orbits of the planets are not perfectly circular but are to varying degrees elliptical, he explained and successfully predicted the motion of Halley's comet, he predicted that the Earth and the other spinning planets should all be slightly flattened in shape. In more modern times Newton's ideas have been used to predict the presence of new planets in our own solar system and more spectacularly to compute the paths of satellites and space vehicles. His theories are triumphantly successful.

Even so, it is worth re-emphasizing that Newton didn't explain what causes the force of gravitation and therefore, you may feel, he didn't really answer the

* If the value you calculated was not close to this, then you should refer to our calculation in Appendix 7.

question we posed at the beginning of the Unit, 'Why do things (planets) move?' Scientists are not able to answer such questions directly. Rather, they attempt to find unifying principles, simple statements which explain seemingly unconnected observations. They attempt to find order in the chaos. In Units 6 and 7 you will meet another unifying idea, the theory of plate tectonics. This relatively simple picture of the Earth's crust connects a jumble of seemingly unconnected geological observations; it unifies. There are many other examples of theories like these; they form the structure of scientific thought and, we hope, the structure of S101.

Aims and Objectives

Apart from Objective 1, which relates to all the terms and concepts used in this Unit, the Objectives of this Unit are related to two general Aims.

The first Aim is the discussion of Newton's theories of motion. You should now be able to:

1 Define correctly, recognize the best definitions of, and distinguish between true and false statements concerning the terms, concepts and principles listed in Table A.

2 Use the terms speed, velocity and acceleration accurately and calculate the values of these quantities for simple examples of motion in a straight line (SAQs 2, 3, 5, 6 and 7).

3 Explain how the concepts of force and mass are scientifically defined (SAQs 8 and 10).

4 State Newton's three laws of motion and use them to explain simple examples of straight-line and orbital motion (SAQs 9, 11, 12, 14, 15 and 16).

5 Differentiate between mass and weight (SAQ 13).

6 State the dependence of the gravitational force of attraction on the masses of the attracted objects and their distance apart, $F = Gm_1m_2/d^2$, and use this equation in simple calculations (SAQ 24).

7 Define density and make simple calculations using the definition (SAQ 25).

8 Describe simple methods of determining the acceleration due to gravity of falling and orbiting objects (Home Experiments).

9 Use the results of such experiments to verify Newton's theory of gravitation (Home Experiments).

10 Define the momentum of an object of known mass and velocity, state the principle of conservation of momentum and use it to explain simple examples of the straight-line motion of two interacting objects (TV 03 and Broadcast Notes).

The second Aim is that by following through the activities contained in the text you should have acquired a number of scientific skills which are of general use. You should be able to:

11 Use Pythagoras's theorem (SAQ 1).

12 Write the units associated with a quantity using positive or negative exponents (SAQ 4).

13 Calculate the percentage error in a reading and in the nth power of a reading (SAQ 17).

14 Combine the errors in two readings to calculate the error in either their product or their ratio (SAQ 23).

15 Choose axes for a graph so that the expression relating two proportional quantities (that is, $y = kx$ or $y^2 = kx^3$, etc.) appears as a straight line (SAQs 19 and 20).

16 Define the gradient of a straight-line graph and identify the constant of proportionality linking the proportional quantities on the axes as being equal to that gradient (SAQs 21 and 22).

17 Represent uncertainties in data by error bars on graphs (Home Analysis).

Appendices

The purpose of these appendices is to take you through the experiments and some calculations in the text. Do *not* use them unless you are unable to perform the experiments or analyses for yourself.

Appendix 1 Falling pebble experiment (Section 5.1)

Each pebble was dropped from the bedroom window of a two-storey house. The height was found by lowering a weighted string to the ground and subsequently measuring the length of the string with a dressmaker's tape. Five readings were taken of the time of fall:

$$\text{height} = 5.15 \pm 0.05\,\text{m}$$

$$\text{time readings:} \quad 1.0, 1.2, 0.8, 0.8, 1.2\,\text{s}$$

$$\text{average time} = (1.0 + 1.2 + 0.8 + 0.8 + 1.2)/5\,\text{s} = 1.0\,\text{s}$$

Assuming that the systematic error is small, the spread of readings should indicate the uncertainty in the average. It is unlikely that the true time-of-fall is higher than our highest reading or lower than our lowest.

So, average time $= 1.0 \pm 0.2\,\text{s}$

that is, an error of 20 per cent

The percentage error in h is approximately 1 per cent and, since it is much smaller than the error in t, we ignore the smaller error in subsequent calculations.

Our best estimate of g_E is:

$$g_E = 2h/t^2 = (2 \times 5.15)/1^2\,\text{m}\,\text{s}^{-2} = 10.3\,\text{m}\,\text{s}^{-2}$$

Our highest acceptable estimate of g_E is:

$$g_E = (2 \times 5.15)/(0.8)^2\text{m}\,\text{s}^{-2}. \approx 16\text{m}\,\text{s}^{-2}$$

Our lowest acceptable estimate of g_E is:

$$g_E = (2 \times 5.15)/(1.2)^2\ \text{m s}^{-2} \approx 7\ \text{m s}^{-2}$$

The data are consistent with g_E lying between 7 and 16 m s^{-2}. Because t is squared and there is a large uncertainty (20 per cent) in its value, the 'best value' of g_E is not in the centre of the range of 'allowable values'. This is not particularly important. Quoting the final result as:

$$g_E = 10 \pm 4\ \text{m s}^{-2}$$

is still a reasonable reflection of the uncertainty in the data. Because of this large error, the best estimate for g_E is given to two significant figures: further precision would be at best meaningless and possibly confusing.

Appendix 2 Stroboscopic determination of g_E (Section 5.2)

Values for the distance travelled were taken from the photograph by laying a clear plastic ruler between the scales. The uncertainty for each reading was subjectively estimated and is presented in Table 2.

TABLE 2

time of fall/s	time^2/s^2	distance fallen/m
0	0	0.000 ± 0.002
0.04	0.0016	0.008 ± 0.003
0.08	0.0064	0.033 ± 0.003
0.12	0.0144	0.073 ± 0.003
0.16	0.0256	0.128 ± 0.003
0.20	0.0400	0.198 ± 0.002
0.24	0.0576	0.285 ± 0.003

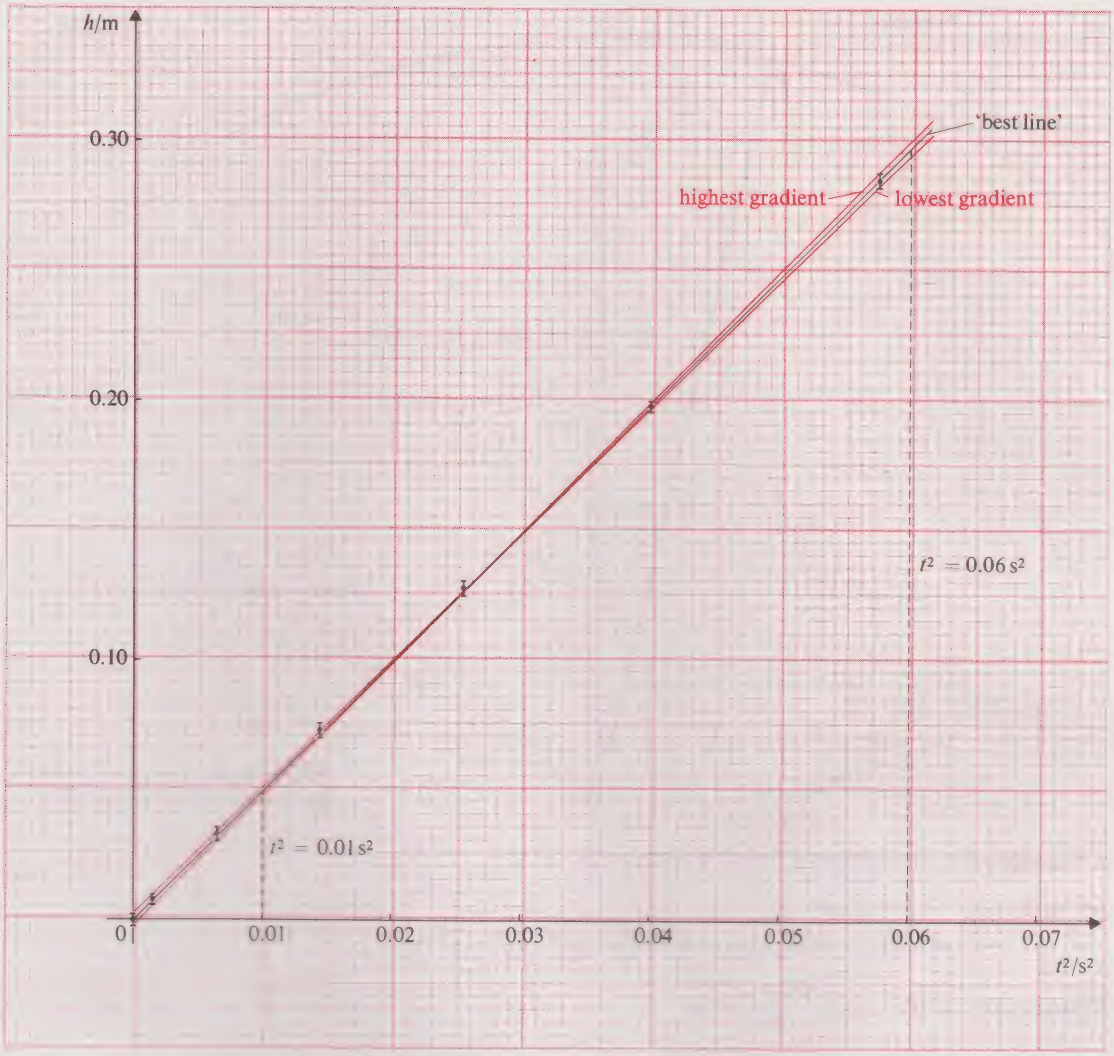


FIGURE 27 The data from Table 2 plotted on a graph with axes h and t^2. Notice that the lines of highest and lowest gradient pass through all the error bars.

The data have been plotted in Figure 27 with the error bars appropriate to each 'point'.

Appendix 3 Stroboscopic determination of g_E (Section 5.2)

The 'best line' and the lines of greatest and least gradient consistent with the error bars are drawn in Figure 27. The gradient for each line has been taken between the points at which it reaches the values $t^2 = 0.01\,\text{s}^2$ and the point $t^2 = 0.06\,\text{s}^2$.

$$\text{Gradient of best line} = (0.297 - 0.050)/0.05\,\text{m s}^{-2} = 4.94\,\text{m s}^{-2}$$

$$\text{Greatest acceptable gradient} = (0.300 - 0.048)/0.05\,\text{m s}^{-2} = 5.04\,\text{m s}^{-2}$$

$$\text{Least acceptable gradient} = (0.294 - 0.050)/0.05\,\text{m s}^{-2} = 4.88\,\text{m s}^{-2}$$

Rounding off these values to an accuracy consistent with the error:

$$\text{gradient} = 4.95 \pm 0.1\,\text{m s}^{-2}$$

$$\text{and } g_E = 2 \times \text{gradient}$$

$$= 9.9 \pm 0.2\,\text{m s}^{-2}$$

Appendix 4 Motion of the Moon (Section 5.3)

The construction used to find the orbital acceleration is shown to the correct scale in Figure 28 (p. 44). *Note that the radius was drawn first* and then the direction of motion at Q (the tangent) was constructed perpendicular to it using a protractor. The distance h, although rather small, was measured with a ruler.

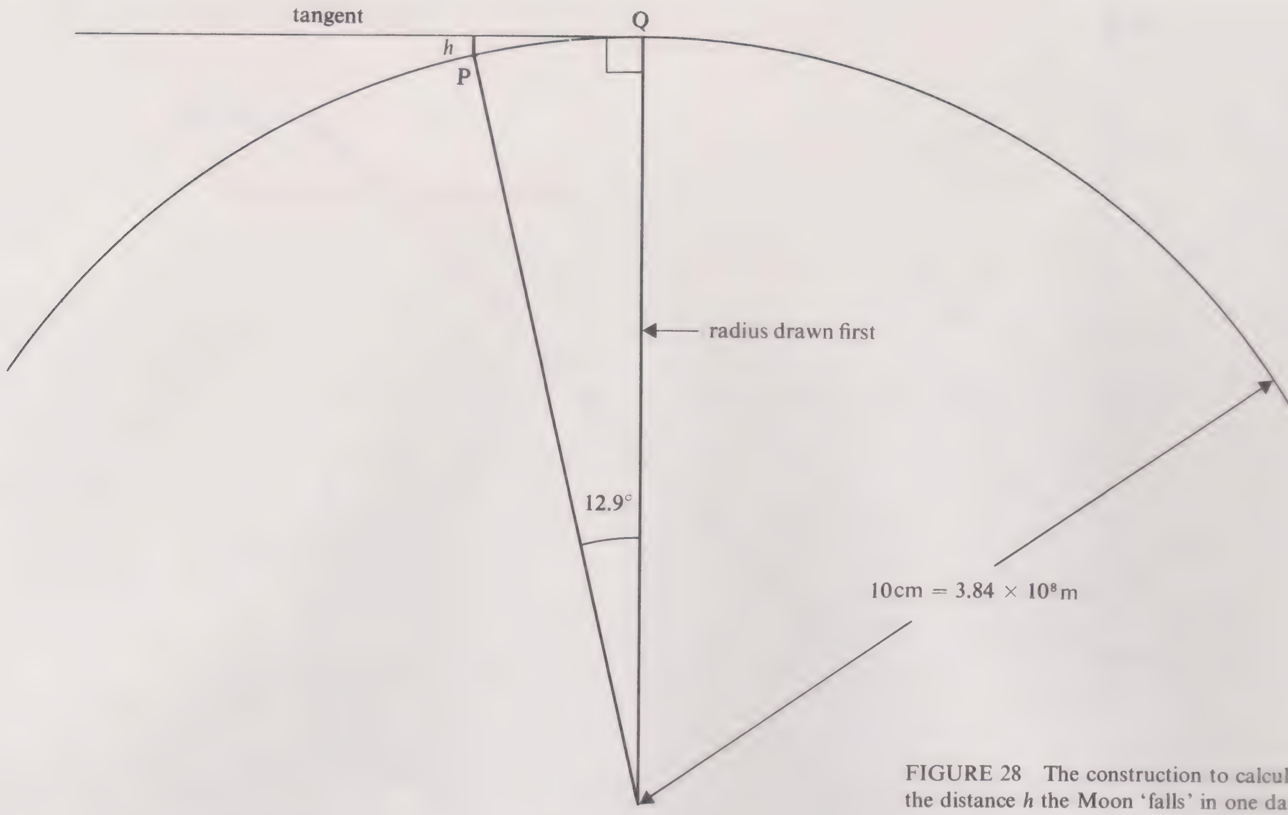


FIGURE 28 The construction to calculate the distance h the Moon 'falls' in one day.

Moon moves through an angle of $(360/28)° \approx 12.9°$ in one day

Measured value of $h = 2.5 \pm 0.3$ mm

$= 0.0025 \pm 0.0003$ m (about 10 per cent uncertainty)

Therefore h (in real life) $= 0.0025 \times \dfrac{(3.84 \times 10^8)}{0.1}$ m

Therefore distance Moon falls in one day is:

$$h = 9.6 \times 10^6 \text{ m } (\pm 10 \text{ per cent})$$
$$t = \text{one day} = 24 \times 3600 \text{ s} = 8.64 \times 10^4 \text{ s}$$

Therefore

$$\begin{aligned}\text{orbital acceleration } a_M &= 2h/t^2 \\ &= (2 \times 9.6 \times 10^6)/(8.64 \times 10^4)^2 \text{ m s}^{-2} \ (\pm 10 \text{ per cent}) \\ &= 2.57 \times 10^{-3} \text{ m s}^{-2} \ (\pm 10 \text{ per cent}) \\ &= (2.6 \pm 0.3) \times 10^{-3} \text{ m s}^{-2}\end{aligned}$$

Appendix 5 Is Newton's prediction of an inverse-square law correct? (Section 5.3)

The data are	$g_E = 9.9 \pm 0.2 \text{ m s}^{-2}$	(our analysis)
	$a_M = (2.6 \pm 0.3) \times 10^{-3} \text{ m s}^{-2}$	(our analysis)
	$R_E = 6.38 \times 10^6$ m	(accepted value)
	$d_M = 3.84 \times 10^8$ m	(accepted value)

Therefore, $d_M/R_E = 3.84 \times 10^8/6.38 \times 10^6$

$(d_M/R_E)^2 = 3623$

Notice that no units are given for the expression $(d_M/R_E)^2$. The ratio of two distances, which should, of course, be measured in the same units, is a dimensionless number.

Percentage error in $g_E = \dfrac{0.2}{9.9} \times 100 \approx 2$ per cent

percentage error in a_M = 10 per cent (from Appendix 4)

Using the statistical theory result,

$$\text{percentage error in } g_E/a_M = \sqrt{[(2)^2 + (10)^2]}\,\%$$
$$= \sqrt{104}\,\% = 10.2\,\%$$
$$\approx 10 \text{ per cent}$$

So, we can neglect the error in g_E in calculating the ratio g_E/a_M (the rule of thumb works!):

$$g_E/a_M = \frac{9.9}{2.6 \times 10^{-3}} \pm 10\%$$
$$= 3\,808 \pm 10 \text{ per cent}$$

Rounding off the numbers,

$$g_E/a_M = 3\,800 \pm 400$$

The value of g_E/a_M is equal to $(d_M/R_E)^2$ to within the accuracy of the experimental data.

Appendix 6 Mass and density of the Earth (Section 6.1)

$$\text{Mass of Earth} = g_E\,R_E{}^2/G$$
$$= (9.9 \pm 0.2) \times (6.38 \times 10^6)^2/(6.67 \times 10^{-11})\,\text{kg}$$
$$= (6.0 \pm 0.1) \times 10^{24}\,\text{kg}$$

$$\text{Average density of Earth} = 3g_E/4G\pi R_E$$
$$= 3 \times (9.9 \pm 0.2)/(4 \times 6.67 \times 10^{-11} \times 3.14)$$
$$\times (6.38 \times 10^6)\,\text{kg}\,\text{m}^{-3}$$
$$= 5\,550 \pm 100\,\text{kg}\,\text{m}^{-3}$$

Appendix 7 Mass and density of the Moon (Section 6.2)

The astronaut drops the hammer from about chest height. Assuming he is 1.9 metres tall,

$$h = (1.5 \pm 0.2) \text{ metres (13 per cent error)}$$

The time of fall was measured on the television programme to be 1.4 s but, because of operator error in the starting and stopping of the watch, we estimate this value could be in error by up to 15 per cent.

So $$t = 1.4\,\text{s} \pm 15 \text{ per cent}$$

and $$t^2 = 1.4^2\,\text{s}^2 \pm 30 \text{ per cent}$$
$$= 2.0\,\text{s}^2 \pm 30 \text{ per cent}$$

Notice that squaring the value of t doubles the percentage error.

$$g_M = 2h/t^2$$
$$= 2 \times 1.5/2.0\,\text{m}\,\text{s}^{-2} \pm \text{error}$$
$$= 1.5\,\text{m}\,\text{s}^{-2} \pm \text{error}$$

$$\text{Percentage error in } g_M = \sqrt{(\%\text{ error in } h)^2 + (\%\text{ error in } t^2)^2}$$
$$= \sqrt{13^2 + 30^2}$$
$$= \sqrt{1\,069}$$
$$\approx 33 \text{ per cent}$$

So $$g_M = 1.5\,\text{m}\,\text{s}^{-2} \pm 33 \text{ per cent}$$
$$= (1.5 \pm 0.5)\,\text{m}\,\text{s}^{-2}$$

Average density of Moon

$$= 3g_M/4G\pi R_M \pm 33 \text{ per cent}$$
$$= 3 \times 1.5/(4 \times 6.67 \times 10^{-11} \times \pi \times 1.74 \times 10^6)\,\text{kg}\,\text{m}^{-3}$$
$$\pm 33 \text{ per cent}$$
$$= 3\,000 \pm 1\,000\,\text{kg}\,\text{m}^{-3}$$

SAQ answers and comments

SAQ 1 200 miles. The right-angled triangle ABC is shown in Figure 29. The sides AC and BC are respectively 120 and 160 miles.

Therefore
$$AB^2 = AC^2 + BC^2$$
$$= (120^2 + 160^2)\ (\text{miles})^2$$
$$AB^2 = 40\,000\ \text{miles}^2$$
$$AB = 200\ \text{miles}$$

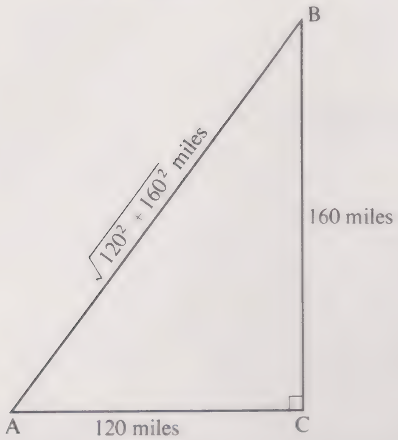


FIGURE 29 The use of Pythagoras's theorem to find the hypotenuse of a right-angled triangle.

SAQ 2 (a) is true, (b) is false.

The velocity of an object is its speed in a given direction. A knowledge of velocity implies a knowledge of speed and direction of motion but a knowledge of speed must be supplemented by a knowledge of direction of motion to define the velocity.

SAQ 3 $5\,\text{m s}^{-1}$ in a northerly direction.

$$\text{Magnitude of velocity} = \text{distance/time}$$
$$= 500/100\ \text{m s}^{-1}$$
$$= 5\ \text{m s}^{-1}$$

SAQ 4 The dimensions are (length) × (time)$^{-2}$, that is:

$$[\text{acceleration}] = [L] \times [T^{-2}] = [LT^{-2}]$$

The SI units are m s^{-2}.

The velocity is the distance travelled in unit time in a given direction. So velocity has dimensions of length/time, that is, (length) × (time)$^{-1}$. The magnitude of the acceleration is the change in velocity per unit time, that is:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

So the dimensions of acceleration are those of velocity/time, that is:

$$(\text{length}) \times (\text{time})^{-1}/\text{time} = (\text{length}) \times (\text{time})^{-2}.$$

The SI units of length are metres and the units of time are seconds.

SAQ 5 The average acceleration is $1/120\,\text{m s}^{-2}$.

The tanker accelerates from rest, and moves in a straight line.

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

The units used must be consistent and so 10 minutes must be written as 600 seconds.

$$\text{average acceleration} = 5\,\text{m s}^{-1}/600\,\text{s}$$
$$= 1/120\,\text{m s}^{-2}$$

SAQ 6 The average acceleration is $(-1/60)\,\text{m s}^{-2}$.

The change in the velocity is negative, that is:

$$\text{new velocity} - \text{old velocity} = -5\,\text{m s}^{-1}$$

So, the acceleration is given by

$$\text{acceleration} = -5\,\text{m s}^{-1}/300\,\text{s}$$
$$= -1/60\,\text{m s}^{-2}$$

The negative sign is important; it indicates that the acceleration is acting in such a way as to reduce the speed (deceleration).

SAQ 7 Assuming they finish the course, all the athletes run round two bends. Even if their speed remains constant, the athletes' direction of motion must change (unless they run into the crowd) and therefore, by definition, there is an acceleration.

SAQ 8 Yes. The Moon follows an approximately circular orbit round the Earth, continually changing its direction of motion and therefore continually accelerating. If a body is accelerating, there must, by Newton's first law of motion, be a force acting on it.

SAQ 9 The chair remains still: it does not accelerate. By Newton's first law there cannot be a net force acting on it. It follows that the floor must be pushing up with a force which exactly balances the downward force applied by the sitter.

SAQ 10 Equation 6 can be used to solve this problem. Suppose the mass of A is one hundred times that of B, that is:

$$m_A = 100\,m_B$$

but
$$m_A a_A = m_B a_B$$

We can substitute the value of $m_A = 100\,m_B$ into the second equation,

$$100\,m_B a_A = m_B a_B$$

Dividing by m_B on both sides,

$$100\,a_A = a_B$$

So the acceleration of B, the trolley and contents with small mass, is much greater than that of A, the trolley and contents with much greater mass.

SAQ 11 The force is 10^6 newtons.

A simple application of Newton's second law is all that is required.

$$\text{force} = 1.2 \times 10^8\,\text{kg} \times (1/120)\,\text{m s}^{-2}$$
$$F = 10^6\,\text{kg m s}^{-2}$$
$$= 10^6\,\text{N}$$

SAQ 12 The car would stop in 20 milliseconds.

To stop the car, the force must be applied in the direction opposite to the direction of motion. A force of 10^6 N will produce, in a car of mass 1 000 kg, an acceleration of:

$$a = F/m$$
$$= 10^6/1\,000\,\text{m s}^{-2}$$
$$= 1 \times 10^3\,\text{m s}^{-2}$$

The original speed of the car is $20\,\text{m s}^{-1}$ and the acceleration is negative (directed against the initial direction of motion):

$$\text{acceleration} = \frac{\text{final speed} - \text{original speed}}{\text{time}}$$

$$-10^3\,\text{m s}^{-2} = \frac{(0 - 20)\,\text{m s}^{-1}}{\text{time} \times \text{s}}$$

$$\text{time} = \frac{-20}{-10^3}\,\text{s}$$
$$= 20 \times 10^{-3}\,\text{s}$$

SAQ 13 No, the accelerations will be different.

The mass of the object is the same at the North Pole as it is at the Equator. So, using Newton's second law,

weight at North Pole = mass × acceleration at North Pole

weight at Equator = mass × acceleration at Equator

If the weights differ by 0.5 per cent, so also will the accelerations.

SAQ 14 The Sun must have a much greater mass than each of the planets.

The force of attraction between the Sun and each planet acts equally on both bodies, but the accelerations produced are very different. From Newton's second law, the relatively small acceleration of the Sun implies that it has a relatively larger mass. In fact the mass of the Sun is about a thousand times greater than that of Jupiter, the largest planet.

SAQ 15 The gun exerts a force on the bullet accelerating it forwards. By Newton's third law, the bullet must exert an equal but opposite force on the gun, accelerating it backwards.

SAQ 16 Although the forces are the same, the bullet has a much smaller mass than the gun. It therefore accelerates more ($a = F/m$).

SAQ 17 Percentage error in l is 2 per cent, in V is 6 per cent.

The percentage error is given by:

$$\text{percentage error} = \text{fractional error} \times 100$$
$$\text{percentage error} = (\text{error/reading}) \times 100$$
$$\text{percentage error in } l = (0.002/0.100) \times 100$$
$$= 2 \text{ per cent}$$

V is found by raising l to the third power ($V = l^3$), and so the percentage error in V is three times the percentage error in l, that is, 6 per cent.

SAQ 18 The strobe is flashing at intervals of 40 ms. Therefore the fourth image of the balls (not counting the first image) records their position at $t = 4 \times 40\,\text{ms} = 0.16\,\text{s}$. From the scales we estimate the distance travelled to lie between 0.125 m and 0.131 m, that is, $h = 0.128 \pm 0.003\,\text{m}$.

SAQ 19 The axes should be V and r^3.

The two quantities which vary in the equation $V = (4/3)\pi r^3$ are V and r. From the equation, V is proportional to r^3, with the constant of proportionality equal to $(4/3)\pi$. A straight-line graph is produced if proportional quantities (V and r^3) are chosen as the axes.

SAQ 20 You should have chosen h and t^2 for the axes.

These are the quantities that are proportional to each other in the equation $h = \frac{1}{2}g_{\text{E}}t^2$.

SAQ 21 The gradient is 1 kg.

$$\text{gradient} = \frac{\text{force at C} - \text{force at A}}{\text{acceleration at C} - \text{acceleration at A}}$$
$$= \frac{(20 - 5)\,\text{N}}{(20 - 5)\,\text{m}\,\text{s}^{-2}}$$
$$= 1\,\text{N/m}\,\text{s}^{-2}$$
$$= 1\,\text{kg}$$

The gradient is the same as it is between points A and B.

SAQ 22 The gradient is $\frac{1}{2}g_{\text{E}}$.

The gradient is the constant of proportionality linking the two quantities plotted on the axes. For this example, h is proportional to t^2 with the constant of proportionality (the gradient) equal to $\frac{1}{2}g_{\text{E}}$.

SAQ 23 The area of the rectangle is $2.0 \pm 0.3\,\text{cm}^2$.

$$\text{area of rectangle} = ab = 2 \times 1\,\text{cm}^2$$
$$= 2\,\text{cm}^2$$
$$\text{percentage error in } a = \frac{0.2}{2.0} \times 100 = 10 \text{ per cent}$$
$$\text{percentage error in } b = \frac{0.1}{1.0} \times 100 = 10 \text{ per cent}$$

From statistical theory,

$$\text{percentage error in } ab = \sqrt{(\%\text{ error in } a)^2 + (\%\text{ error in } b)^2}$$
$$= \sqrt{10^2 + 10^2}$$
$$= \sqrt{200}$$
$$\approx 14 \text{ per cent}$$

$$14 \text{ per cent of } 2.0\,\text{cm}^2 = \frac{14 \times 2.0}{100}\,\text{cm}^2 = 0.28\,\text{cm}^2$$
$$\approx 0.3\,\text{cm}^2$$

So, area of rectangle $ab = 2.0 \pm 0.3\,\text{cm}^2$.

SAQ 24 $G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$.

The equation $F = Gm_1m_2/d^2$ can be rearranged to provide an expression for G by multiplying both sides by d^2/m_1m_2, that is:

$$F \times \frac{d^2}{m_1m_2} = G \times \frac{m_1m_2}{d^2} \times \frac{d^2}{m_1m_2} = G$$

Therefore, $$G = \frac{Fd^2}{m_1m_2}$$

From this experiment $F \approx 6.67 \times 10^{-9}\,\text{N}$, $d \approx 0.1\,\text{m}$ and $m_1 = m_2 = 1\,\text{kg}$,

So $$G = \frac{6.67 \times 10^{-9} \times (0.1)^2\,\text{N}\,\text{m}^2}{1 \times 1\,\text{kg}^2}$$
$$= 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$$

SAQ 25 The density of lignum vitae is $1.3\,\text{g}\,\text{cm}^{-3}$.

Density is mass per unit volume.

So $$\text{density of lignum vitae} = \frac{\text{mass}}{\text{volume}} = \frac{13\,\text{g}}{10\,\text{cm}^3}$$
$$= 1.3\,\text{g}\,\text{cm}^{-3}$$

Incidentally, lignum vitae is one of the most dense woods known. Typically, woods have a density slightly less than that of water, $1.0\,\text{g}\,\text{cm}^{-3}$.

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